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CHAPTER I

Characteristics of a Controlled Quality

1. *What is the Problem of Control?*

What is the problem of control of quality of manufactured product? To answer this question, let us put ourselves in the position of a manufacturer turning out millions of the same kind of thing every year. Whether it be lead pencils, chewing gum, bars of soap, telephones, or automobiles, the problem is much the same. He sets up a standard for the quality of a given kind of product. He then tries to make all pieces of product conform with this standard. Here his troubles begin. For him standard quality is a bull's-eye, but like a marksman shooting at a bull's-eye, he often misses. As is the case in everything we do, unknown or chance causes exert their influence. The problem then is: how much may the quality of a product vary and yet be controlled? In other words, how much variation should we leave to chance?

To make a thing the way we want to make it is one popular conception of control. We have been trying to do this for a good many years and we see the fruition of this effort in the marvelous industrial development around us. We are sold on the idea of applying scientific principles. However, a change is coming about in the principles themselves and this change gives us a new concept of control.

A few years ago we were inclined to look forward to the time when a manufacturer would be able to do just what he wanted to do. We shared the enthusiasm of Pope when he said "All chance is but direction thou canst not see", and we looked forward to the time when we would see that direction. In other words, emphasis was laid on the *exactness* of physical laws. Today, however, the emphasis is placed elsewhere as is indicated by the following quotation from a recent issue, July, 1927, of the journal *Engineering*:

> Today the mathematical physicist seems more and more inclined to the opinion that each of the so-called laws of nature is essentially statistical, and that all our equations and theories can do, is to provide us with a series of orbits of varying probabilities.

The breakdown of the orthodox scientific theory which formed the basis of applied science in the past necessitates the introduction of certain new concepts into industrial development. Along with this change must come a revision in our ideas of such things as a controlled product, an economic standard of quality, and the method of detecting lack of control or those variations which should not be left to chance.

Realizing, then, the statistical nature of modern science, it is but logical for the manufacturer to turn his attention to the consideration of available ways and means of handling statistical problems. The necessity for doing this is pointed out in the recent book [1] on the application of statistics in mass production, by Becker, Plaut, and Runge. They say:

> It is therefore important to every technician who is dealing with problems of manufacturing control to know the laws of statistics and to be able to apply them correctly to his problems.

Another German writer, K. H. Daeves, in writing on somewhat the same subject says:

> Statistical research is a logical method for the control of operations, for the research engineer, the plant superintendent, and the production executive.[2]

The problem of control viewed from this angle is a comparatively new one. In fact, very little has been written on the subject. Progress in modifying our concept of control has been and will be comparatively slow. In the first place,

[1] *Anwendungen der Mathematischen Statistik auf Probleme der Massenfabrikation*, Julius Springer, Berlin, 1927.

[2] "The Utilization of Statistics," *Testing*, March, 1924.

it requires the application of certain modern physical concepts; and in the second place, it requires the application of statistical methods which up to the present time have been for the most part left undisturbed in the journals in which they appeared. This situation is admirably summed up in the January, 1926 issue of *Nature* as follows:

> A large amount of work has been done in developing statistical methods on the scientific side, and it is natural for anyone interested in science to hope that all this work may be utilized in commerce and industry. There are signs that such a movement has started, and it would be unfortunate indeed if those responsible in practical affairs fail to take advantage of the improved statistical machinery now available.

2. *Nature of Control*

Let us consider a very simple example of our inability to do exactly what we want to do and thereby illustrate two characteristics of a controlled product.

Write the letter *a* on a piece of paper. Now make another *a* just like the first one; then another and another until you have a series of *a*'s, *a*, *a*, *a*, *a*, ... You try to make all the *a*'s alike but you don't; you can't. You are willing to accept this as an empirically established fact. But what of it? Let us see just what this means in respect to control. Why can we not do a simple thing like making all the *a*'s just alike? Your answer leads to a generalization which all of us are perhaps willing to accept. It is that there are many causes of variability among the *a*'s: the paper was not smooth, the lead in the pencil was not uniform, and the unavoidable variability in your external surroundings reacted upon you to introduce variations in the *a*'s. But are these the only causes of variability in the *a*'s? Probably not.

We accept our human limitations and say that likely there are many other factors. If we could but name all the reasons why we cannot make the *a*'s alike, we would most assuredly have a better understanding of a certain part of nature than we now have. Of course, this conception of what it means to be able to do what we want to do is not new; it does not belong exclusively to any one field of human thought; it is commonly accepted.

The point to be made in this simple illustration is that we are limited in doing what we want to do; that to do what we set out to do, even in so simple a thing as making *a*'s that are alike, requires almost infinite knowledge compared with that which we now possess. It follows, therefore, since we are thus willing to accept as axiomatic that we cannot do what we want to do and cannot hope to understand why we cannot, that we must also accept as axiomatic that a controlled quality will not be a constant quality. Instead, a controlled quality must be a *variable* quality. This is the first characteristic.

But let us go back to the results of the experiment on the *a*'s and we shall find out something more about control. Your *a*'s are different from my *a*'s; there is something about your *a*'s that makes them yours and something about my *a*'s that makes them mine. True, not all of your *a*'s are alike. Neither are all of my *a*'s alike. Each group of *a*'s varies within a certain range and yet each group is distinguishable from the others. This distinguishable and, as it were, constant variability *within limits* is the second characteristic of control.

3. *Definition of Control*

For our present purpose *a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future. Here it is understood that prediction within limits means that we can state, at least approximately, the probability that the observed phenomenon will fall within the given limits.*

In this sense the time of the eclipse of the sun is a predictable phenomenon. So also is the distance covered in successive intervals of time by a freely falling body. In fact, the prediction in such cases is extremely precise. It is an entirely different matter, however, to predict the expected length of life of an individual at a given age; the velocity of a molecule at a given instant of time; the breaking strength of a steel wire of known cross section; or numerous other phenomena of like character. In fact, a prediction of the type illustrated by forecasting the time of an eclipse of the sun is almost the exception rather than the rule in scientific and industrial work.

In all forms of prediction an element of chance enters. The specific problem which concerns us at the present moment is the formulation of a scientific basis for prediction, taking into account the element of chance, where, for the purpose of our discussion, *any unknown cause of a phenomenon will be termed a chance cause.*

CHAPTER II

Scientific Basis for Control

1. *Three Important Postulates*

What can we say about the future behavior of a phenomenon acting under the influence of unknown or chance causes? I doubt that, in general, we can say anything. For example, let me ask: "What will be the price of your favorite stock thirty years from today?" Are you willing to gamble much on your powers of prediction in such a case? Probably not. However, if I ask: "Suppose you were to toss a penny one hundred times, thirty years from today, what proportion of heads would you expect to find?", your willingness to gamble on your powers of prediction would be of an entirely different order than in the previous case.

The recognized difference between these two situations leads us to make the following simple postulate:

> *Postulate 1—All chance systems of causes are not alike in the sense that they enable us to predict the future in terms of the past.*

Hence, if we are to be able to predict the quality of product even within limits, we must find some criterion to apply to observed variability in quality to determine whether or not the cause system producing it is such as to make future predictions possible.

Perhaps the natural course to follow is to glean what we can about the workings of unknown chance causes which are generally acknowledged to be controlled in the sense that they permit of prediction within limits. Perhaps no better examples could be considered than length of human life and molecular motion. It might appear that nothing is more uncertain than life itself, unless perhaps it be molecular motion. Yet there is something certain about these uncertainties. In the laws of mortality and distribution of molecular displacement, we find some of the essential characteristics of control within limits.

A. *Law of Mortality*

The date of death always has seemed to be fixed by chance even though great human effort has been expended in trying to rob chance of this prerogative. We come into this world and from that very instant on are surrounded by causes of

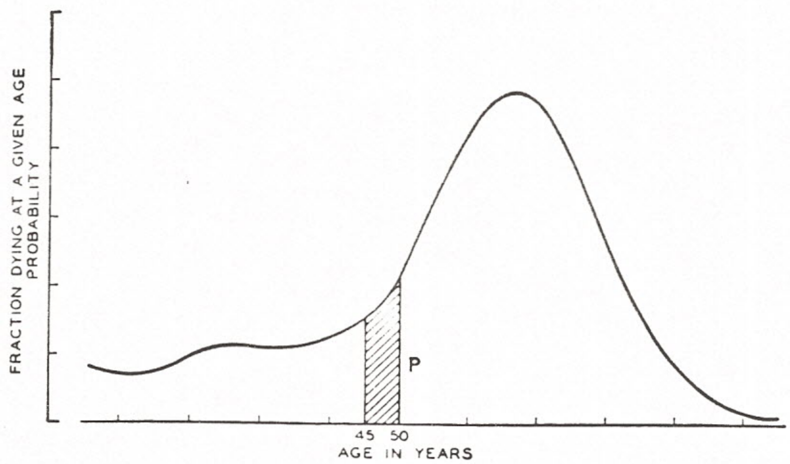

FIG. 1.—LAW OF MORTALITY—LAW OF FLUCTUATIONS CONTROLLED WITHIN LIMITS.

death seeking our life. Who knows whether or not death will overtake us within the next year? If it does, what will be the cause? These questions we cannot answer. Some of us are to fall at one time from one cause, others at another time from another cause. In this fight for life we see then the element of uncertainty and the interplay of numerous unknown or chance causes.

However, when we study the effect of these chance causes in producing deaths in large groups of individuals, we find some indication of a controlled condition. We find that this hidden host of causes produce deaths at an average rate which does not differ much over long periods of time. From such observations we are led to believe that, as we approach the condition of homogeneity of population and surroundings, we approach what is customarily termed a "Law of Mortality" such as indicated schematically in Fig. 1. In other words, we believe that in the limiting case of homogeneity the causes of death function so as to make the probability of dying within given age limits, such as forty-five to fifty, constant. That is, we believe these causes are controlled. In other words, we assume the existence of a kind of statistical equilibrium among the effects of an unknown system of chance causes expressible in the assumption that the probability of dying within a given age limit, under the assumed conditions, is an objective and constant reality.

B. *Molecular Motion*

Just about a century ago, in 1827 to be exact, an English botanist, Brown, saw something through his microscope that caught his interest. It was motion going on among the suspended particles almost as though they were alive. In a way it resembled the dance of dust particles in sunlight, so familiar to us, but this dance differed from that of the dust particles in important respects,—for example, adjacent particles seen under the microscope did not necessarily move in even approximately the same direction, as do adjacent dust particles suspended in the air.

Watch such motion for several minutes. So long as the temperature remains constant, there is no change. Watch it for hours, the motion remains characteristically the same. Watch it for days, we see no difference. Even particles suspended in liquids enclosed in quartz crystals for thousands of years show exactly the same kind of motion. Therefore, to the best of our knowledge there is remarkable permanence to this motion. Its characteristics remain constant. Here we certainly find a remarkable degree of constancy exhibited by a chance system of causes.

Suppose we follow the motion of one particle to get a better picture of this constancy. This has been done for us by several investigators, notably Perrin. In such an experiment he noted the position of a particle at the end of equal intervals of time, Fig. 2. He found that the direction of this motion observed in one interval differed in general from that in the next succeeding interval; that the direction of the motion

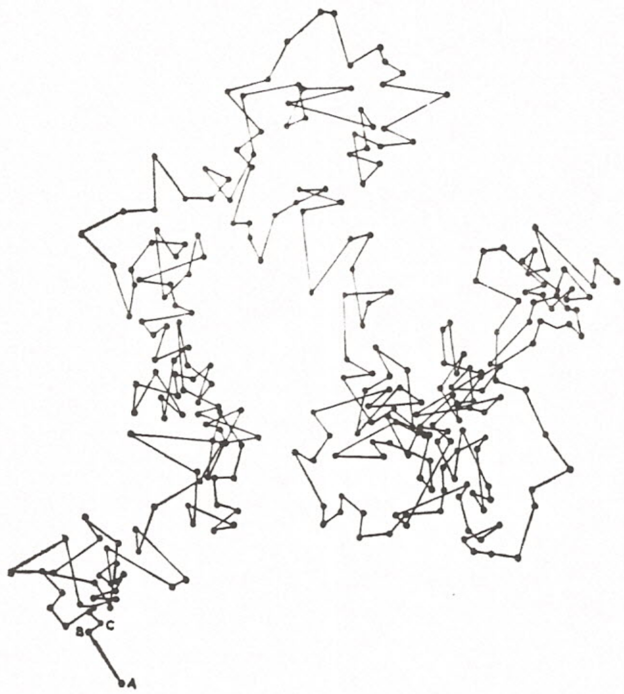

FIG. 2.—A CLOSE-UP OF MOLECULAR MOTION APPEARING ABSOLUTELY IRREGULAR, YET CONTROLLED WITHIN LIMITS.

presents what we instinctively call absolute irregularity. Let us ask ourselves certain questions about this motion.

Suppose we fix our attention on the particle at the point A. What made it move to B in the next interval of time? Of course we answer by saying that a particle moves at a given instant in a given direction, say AB, because the resultant force of the molecules hitting it in a plane perpendicular to this direction from the side away from B is greater than tha on the side toward B; but at any given instant of time ther is no way of telling what molecules are engaged in giving such motion. We do not even know how many molecules ar taking part. Do what we will, so long as the temperature kept constant, we cannot change this motion in a given systen It cannot be said, for example, when the particle is at the poin B that during the next interval of time it will move to C We can do nothing to control the motion in the matter of dis placement or in the matter of the direction of this displacemen

Let us consider either the x or y components of the segment of the paths. Within recent years we find abundant evidenc indicating that these displacements appear to be distribute about zero in accord with what is called the normal law.[1]

Such evidence as that provided by the law of mortalit and the law of distribution of molecular displacements leads u to assume that there exist in nature phenomena controlled by systems of chance causes such that the probability dy of the magnitude X of a characteristic of some such phenomenon falling within the interval X to $X + dX$ is expressible as a function f of the quantity X and certain parameters represente symbolically in the equation

$$dy = f(X, \lambda_1, \lambda_2, \ldots, \lambda_m)dX, \quad (2)$$

where the λ's denote the parameters. Such a system of causes we shall term *constant* because the probability dy is independent of time. We shall take as our second postulate:

> *Postulate 2—Constant systems of chance causes do exist in nature.*

To say that such systems of causes exist in nature, however, is one thing; to say that such systems of causes exist in a

[1] That is to say, if x represents the deviation from the mean displacement, zero in this case, the probability dy of x lying within the range x to $x + dx$ is given by

$$dy = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} dx, \quad (1)$$

where σ is the root mean square deviation.

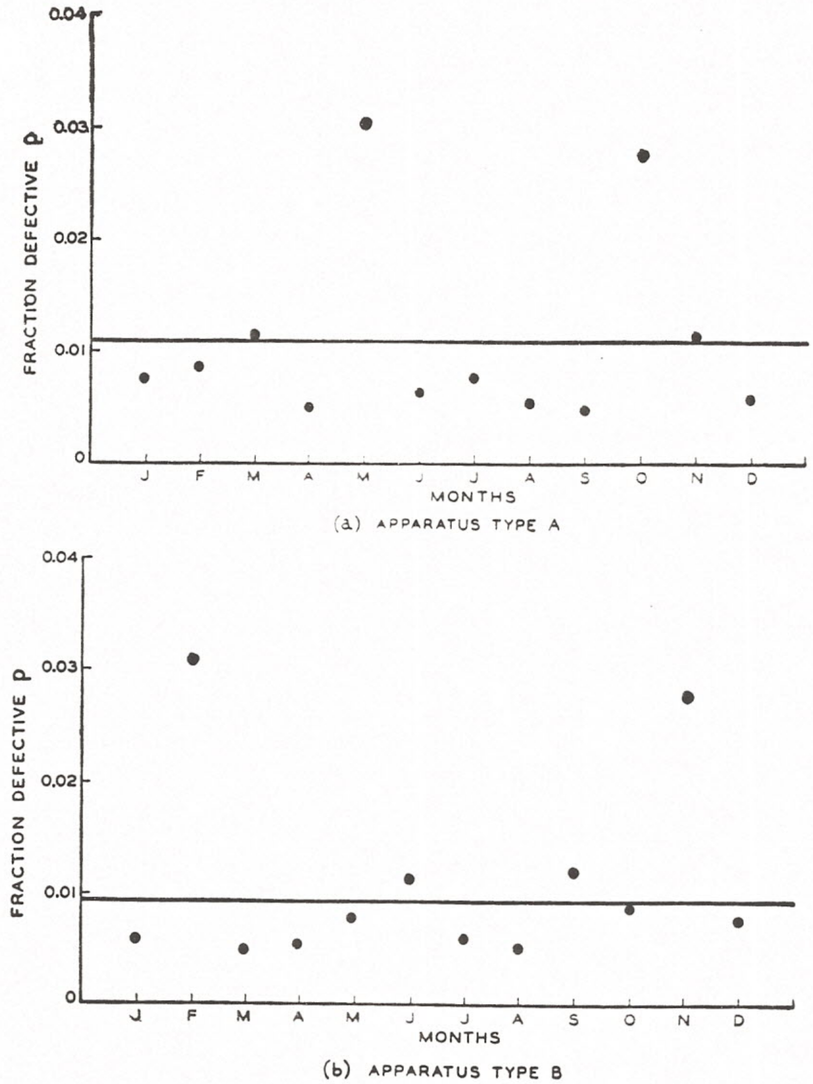

FIG. 3.—SHOULD THESE VARIATIONS BE LEFT TO CHANCE?

production process is quite another thing. Today we have abundant evidence of the existence of such systems of causes in the production of telephone equipment. The practical situation, however, is that in the majority of cases there are unknown causes of variability in the quality of a product which do not belong to a constant system. This fact was discovered very early in the development of control methods, and these causes were called *assignable*. The question naturally arose as to whether it was possible, in general, to find and eliminate such causes. Less than ten years ago it seemed reasonable to assume that this could be done. Today we have abundant evidence to justify this assumption. We shall, therefore, adopt as our third postulate:

Postulate 3—Assignable causes of variation may be found and eliminated.

Hence, to secure control, the manufacturer must seek to find and eliminate assignable causes. In practice, however, he has the difficulty of judging from an observed set of data whether or not assignable causes are present. A simple illustration will make this point clear.

2. *When do Fluctuations Indicate Trouble?*

In many instances the quality of the product is measured by the fraction non-conforming to engineering specifications or, as we say, the fraction defective. Table 1 gives for a period of twelve months the observed fluctuations in this fraction for two kinds of product designated here as Type A and Type B. For each month we have the sample size n, the number defective n_1 and the fraction $p = \frac{n_1}{n}$. We can better visualize the extent of these fluctuations in fraction defective by plotting the data as in Fig. 3-*a* and Fig. 3-*b*.

What we need is some yardstick to detect in such variations any evidence of the presence of assignable causes. Can we find such a yardstick? Experience of the kind soon to be considered indicates that we can. It leads us to conclude that it is feasible to establish criteria useful in detecting the presence of assignable causes of variation or, in other words, criteria which when applied to a set of observed values will indicate whether or not it is reasonable to believe that the causes of variability should be left to chance. Such criteria are basic to any method of securing control within limits. Let us, therefore, consider them critically. It is too much to expect that the criteria will be infallible. We are amply rewarded if they appear to work in the majority of cases.

Generally speaking, the criteria are of the nature of limits derived from past experience showing within what range the fluctuations in quality should remain, if they are to be left to chance. For example, when such limits are placed on the fluctuations in the qualities shown in Fig. 3, we find, as shown in Fig. 4, that in one case two points fall outside the limits and in the other case no point falls outside the limits.

TABLE 1.—FLUCTUATIONS IN QUALITY OF TWO MANUFACTURED PRODUCTS

Apparatus Type A				Apparatus Type B			
Month	Number Inspected n	Number Defective n_1	Fraction Defective $p=\frac{n_1}{n}$	Month	Number Inspected n	Number Defective n_1	Fraction Defective $p=\frac{n_1}{n}$
Jan.	527	4	0.0076	Jan.	169	1	0.0059
Feb.	610	5	0.0082	Feb.	99	3	0.0303
March	428	5	0.0117	March	208	1	0.0048
April	400	2	0.0050	April	196	1	0.0051
May	498	15	0.0301	May	132	1	0.0076
June	500	3	0.0060	June	89	1	0.0112
July	395	3	0.0076	July	167	1	0.0060
Aug.	393	2	0.0051	Aug.	200	1	0.0050
Sept.	625	3	0.0048	Sept.	171	2	0.0117
Oct.	465	13	0.0280	Oct.	122	1	0.0082
Nov.	446	5	0.0112	Nov.	107	3	0.0280
Dec.	510	3	0.0059	Dec.	132	1	0.0076
Average	483.08	5.25	0.0109	Average	149.33	1.42	0.0095

Upon the basis of the use of such limits, we look for trouble in the form of assignable causes in one case but not in the other.

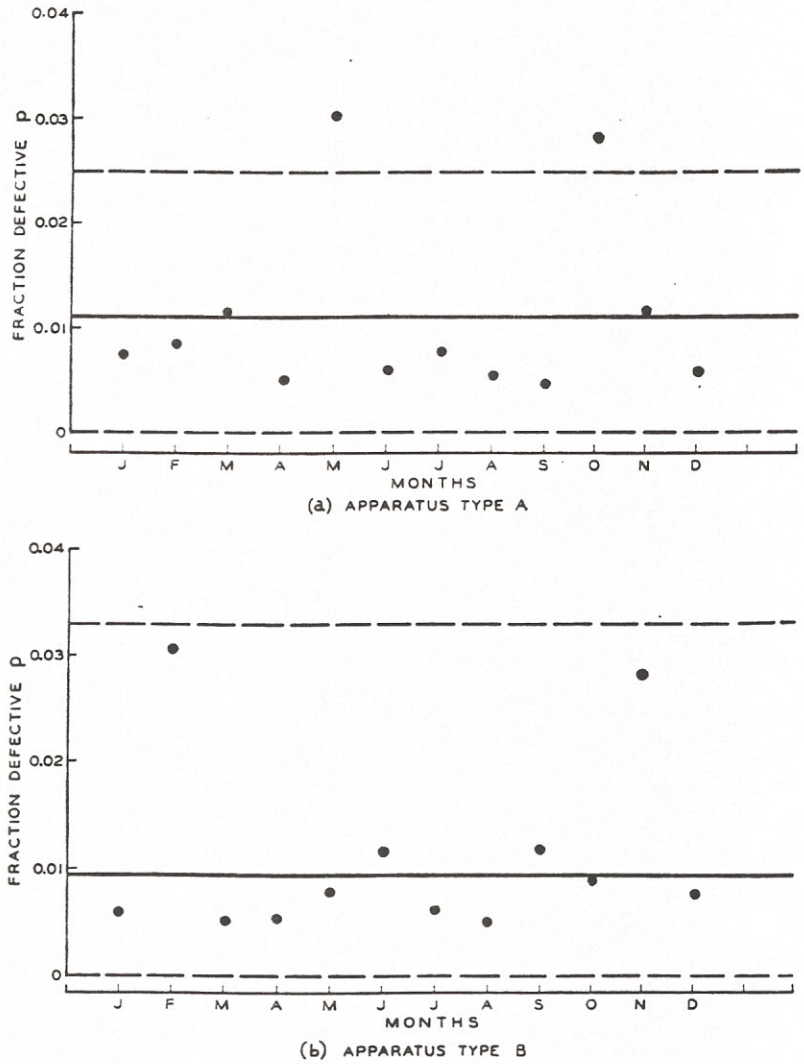

FIG. 4.—SHOULD THESE VARIATIONS BE LEFT TO CHANCE?
a—" No." *b*—" Yes."

However, the question remains: Should we expect to be able to find and eliminate causes of variability only when deviations fall outside the limits? First, let us see what statistical theory has to say in answer to this question.

Upon the basis of Postulate 3, it follows that we can find and remove causes of variability until the remaining system of causes is constant or until we reach that state where the probability that the deviations in quality remain within any two fixed limits (Fig. 5) is constant. However, this assumption alone does not tell us that there are certain limits within which all observed values of quality should remain provided the causes cannot be found and eliminated. In fact, as long as

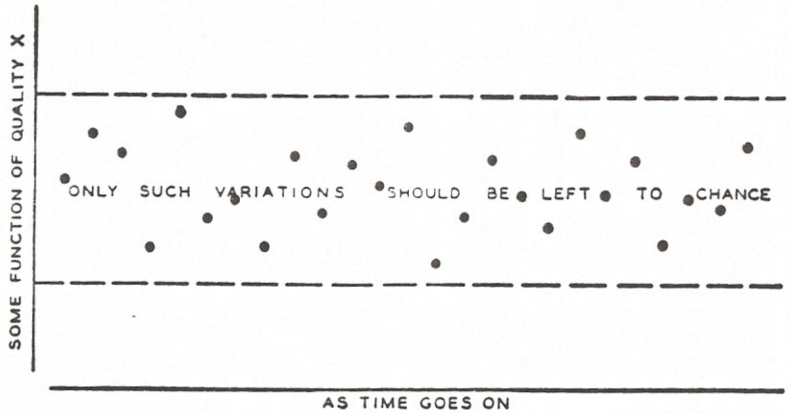

Fig. 5.—Judgment Plus Modern Statistical Machinery Makes Possible the Establishment of Such Limits

the limits are set so that the probability of falling within the limits is less than unity, we may always expect a certain percentage of observations to fall outside the limits even though the system of causes be constant. In other words, the acceptance of this assumption gives us a right to believe that there is an objective state of control within limits but in itself it does not furnish a practical criterion for determining when variations in quality, such as those indicated in Fig. 3, should be left to chance.

Furthermore, we may say that mathematical statistics as such does not give us the desired criterion. What does this situation mean in plain everyday engineering English? Simply this: such criteria, if they exist, cannot be shown to exist by any theorizing alone, no matter how well equipped the theorist is in respect to probability or statistical theory. We see in this situation the long recognized dividing line between theory and practice. The available statistical machinery referred to by the magazine *Nature* is, as we might expect, not an end in itself but merely a means to an end. In other words, the fact that the criterion which we happen to use has a fine ancestry of highbrow statistical theorems does not justify its use. Such justification must come from empirical evidence that it works. As the practical engineer might say, the proof of the pudding is in the eating. Let us therefore look for the proof.

3. *Evidence that Criteria Exist for Detecting Assignable Causes*

A. Fig. 6 shows the results of one of the first large scale experiments to determine whether or not indications given by

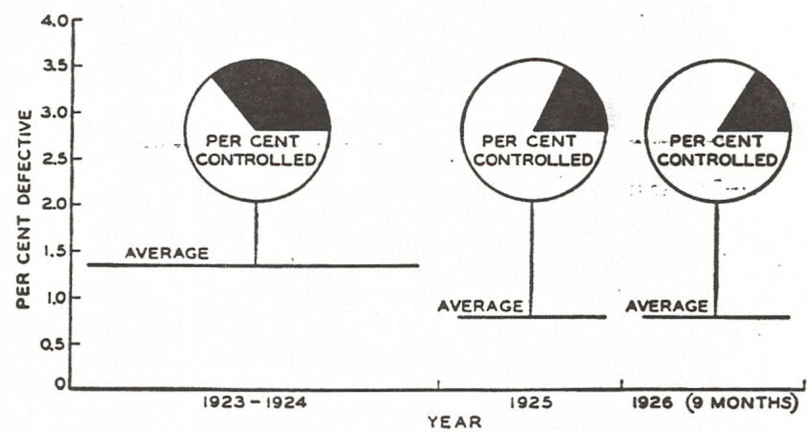

FIG. 6.—EVIDENCE OF IMPROVEMENT IN QUALITY WITH APPROACH TO CONTROL.

such a criterion applied to quality measured in terms of fraction defective were justified by experience. About thirty typical items used in the telephone plant and produced in lots running into the millions per year were made the basis for this study. As shown in this figure, during 1923–24 these items showed

68 per cent control about a relatively low average of 1.4 per cent defective.[1] However, as the assignable causes, indicated by deviations in the observed monthly fraction defective falling outside of control limits, were found and eliminated, the quality of product approached the state of control as indicated by an increase of from 68 per cent to 84 per cent control by the latter part of 1926. At the same time the quality improved; in 1923–24 the average per cent defective was 1.4 per cent, whereas by 1926 this had been reduced to 0.8 per cent. Here we get some typical evidence that, in general, as the assignable causes are removed, the variations tend to fall more nearly within the limits as indicated by an increase from 68 per cent to 84 per cent. Such evidence is, of course, one sided. It shows that when points fall outside the limits, experience indicates that we can find assignable causes, but it does not indicate that when points fall within such limits, we cannot find causes of variability. However, this kind of evidence is provided by the following two typical illustrations.

B. In the production of a certain kind of equipment, considerable cost was involved in securing the necessary electrical insulation by means of materials previously used for that purpose. A research program was started to secure a cheaper material. After a long series of preliminary experiments, a tentative substitute was chosen and an extensive series of tests of insulation resistance were made on this material, care being taken to eliminate all known causes of variability. Table 2 gives the results of 204 observations of resistance in megohms taken on as many samples of the proposed substitute material. Reading from top to bottom beginning at the left column and continuing throughout the table gives the order in which the observations were made. The question is: "Should such variations be left to chance?"

No *a priori* reason existed for believing that the measurements forming one portion of this series should be different from those in any other portion. In other words, there was

[1] Jones, R. L., "Quality of Telephone Materials," *Bell Telephone Quarterly*, June, 1927.

no rational basis for dividing the total set of data into groups of a given number of observations except that it was reasonable to believe that the system of causes might have changed from day to day as a result of changes in such things as atmospheric conditions, observers, and materials. In general, if such changes are to take place, we may readily detect their effect if we divide the total number of observations into comparatively small subgroups. In this particular instance, the size of the subgroup was taken as four and the black dots in Fig. 7-*a* show the successive averages of four observations in the order in which thev were taken. The dotted lines are the

TABLE 2.—ELECTRICAL RESISTANCE OF INSULATION IN MEGOHMS—SHOULD SUCH VARIATIONS BE LEFT TO CHANCE?

5,045	4,635	4,700	4,650	4,640	3,940	4,570	4,560	4,450	4,500	5,075	4,500
4,350	5,100	4,600	4,170	4,335	3,700	4,570	3,075	4,450	4,770	4,925	4,850
4,350	5,450	4,110	4,255	5,000	3,650	4,855	2,965	4,850	5,150	5,075	4,930
3,975	4,635	4,410	4,170	4,615	4,445	4,160	4,080	4,450	4,850	4,925	4,700
4,290	4,720	4,180	4,375	4,215	4,000	4,325	4,080	3,635	4,700	5,250	4,890
4,430	4,810	4,790	4,175	4,275	4,845	4,125	4,425	3,635	5,000	4,915	4,625
4,485	4,565	4,790	4,550	4,275	5,000	4,100	4,300	3,635	5,000	5,600	4,425
4,285	4,410	4,340	4,450	5,000	4,560	4,340	4,430	3,900	5,000	5,075	4,135
3,980	4,065	4,895	2,855	4,615	4,700	4,575	4,840	4,340	4,700	4,450	4,190
3,925	4,565	5,750	2,920	4,735	4,310	3,875	4,840	4,340	4,500	4,215	4,080
3,645	5,190	4,740	4,375	4,215	4,310	4,050	4,310	3,665	4,840	4,325	3,690
3,760	4,725	5,000	4,375	4,700	5,000	4,050	4,185	3,775	5,075	4,665	5,050
3,300	4,640	4,895	4,355	4,700	4,575	4,685	4,570	5,000	5,000	4,615	4,625
3,685	4,640	4,255	4,090	4,700	4,700	4,685	4,700	4,850	4,770	4,615	5,150
3,463	4,895	4,170	5,000	4,700	4,430	4,430	4,440	4,775	4,570	4,500	5,250
5,200	4,790	3,850	4,335	4,095	4,850	4,300	4,850	4,500	4,925	4,765	5,000
5,100	4,845	4,445	5,000	4,095	4,850	4,690	4,125	4,770	4,775	4,500	5,000

limits within which experience has shown that these observations should fall, taking into account the size of the sample, provided the variability should be left to chance. Several of the observed values lie outside these limits. This was taken as an indication of the existence of causes of variability which could be found and eliminated.

Further research was instituted at this point to find these causes of variability. Several were found, and after these had been eliminated another series of observed values gave the results indicated in Fig. 7-*b*. Here we see that all of the points lie within the limits. We assumed, therefore, upon the basis of this test, that it was not feasible for research to go much further in eliminating causes of variability. Because of

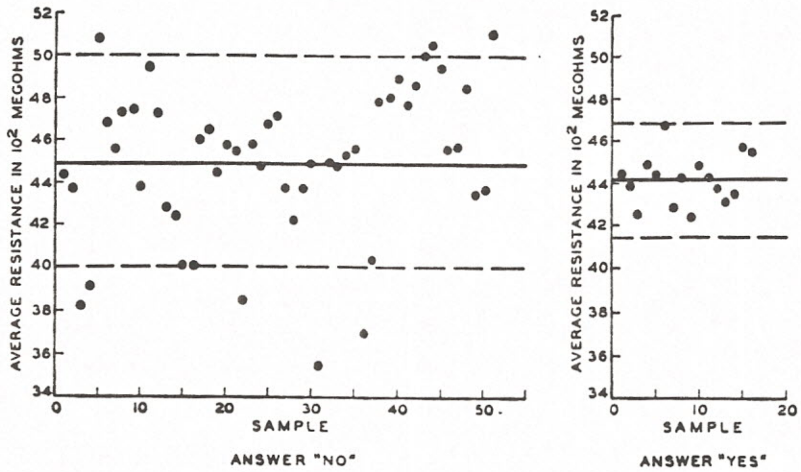

FIG. 7.—SHOULD THESE VARIATIONS BE LEFT TO CHANCE?

the importance of this particular experiment, however, considerably more work was done, but it failed to reveal causes of variability. Here then is a typical case where the criterion indicates when variability should be left to chance.

C. Suppose now that we take another illustration where it is reasonable to believe that almost everything humanly possible has been done to remove the assignable causes of variation in a set of data. Perhaps the outstanding series of observations of this type is that given by Millikan in his famous measurement of the charge on an electron. Treating his data in a manner similar to that indicated above, we get the results shown in Fig. 8. All of the points are within the dotted limits. Hence the indication of the test is consistent with the accepted conclusion that those factors which need not be left to chance had been eliminated before this particular set of data were taken.

4. *Rôle Played by Statistical Theory*

It may appear thus far that mathematical statistics plays a relatively minor rôle in laying a basis for economic control of quality. Such, however, is not the case. In fact, a central concept in engineering work today is that almost every physical property is a *statistical distribution*. In other words, an observed

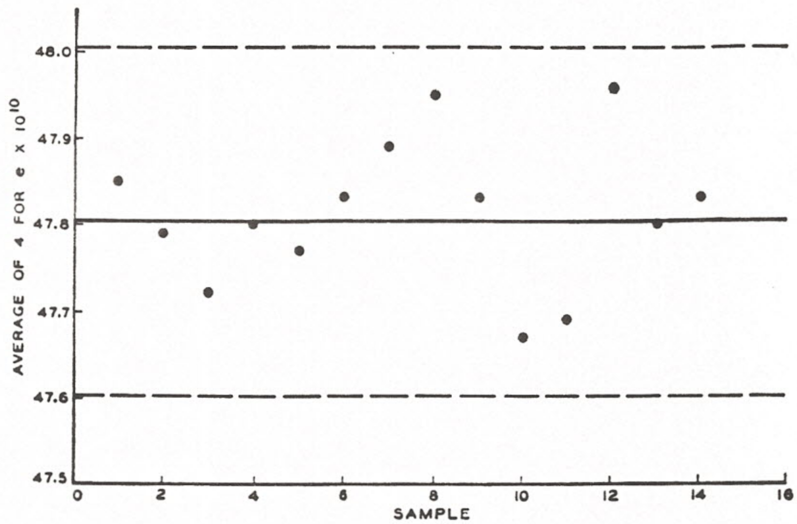

FIG. 8.—VARIATIONS THAT SHOULD BE LEFT TO CHANCE—DOES THE CRITERION WORK? "YES."

set of data constitutes a sample of the effects of unknown chance causes. It is at once apparent, therefore, that sampling theory should prove a valuable tool in testing engineering hypotheses. Here it is that much of the most recent mathematical theory becomes of value, particularly in analysis involving the use of comparatively small numbers of observations.

Let us consider, for example, some property such as the tensile strength of a material. If our previous assumptions are justified, it follows that, after we have done everything we can to eliminate assignable causes of variation, there will still remain a certain amount of variability exhibiting the state of control. Let us consider an extensive series of data recently published by a member of the Forest Products Laboratories,[1] Fig. 9. Here we have the results of tests for modulus of rupture on 1,304 small test specimens of Sitka spruce, the kind of material used extensively in aeroplane propellers

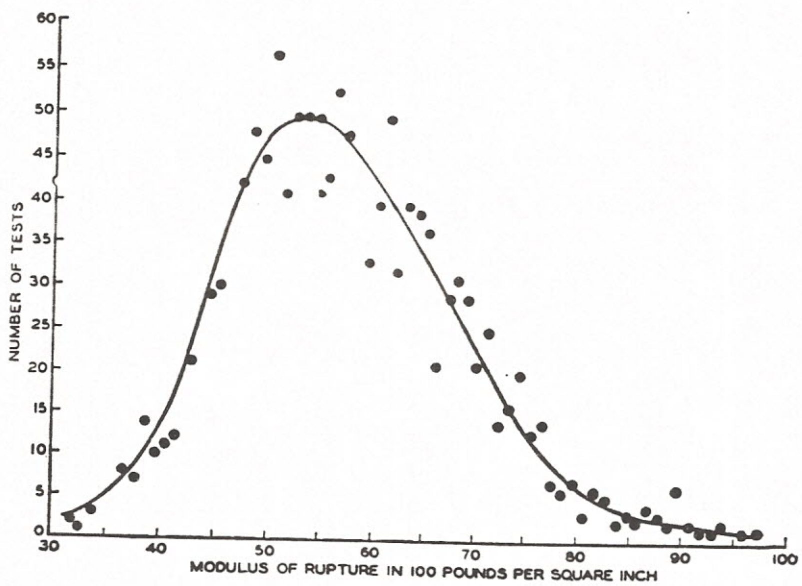

FIG. 9.—VARIABILITY IN MODULUS OF RUPTURE OF CLEAR SPECIMENS OF GREEN SITKA SPRUCE TYPICAL OF THE STATISTICAL NATURE OF PHYSICAL PROPERTIES.

during the War. The wide variability is certainly striking. The curve is an approximation to the distribution function for this particular property representing what is at least approximately a state of control. The importance of going from the sample to the smooth distribution is at once apparent and in this case a comparatively small amount of refinement in statistical machinery is required.

[1] Newlin, J. A., *Proceedings of the American Society of Civil Engineers*, September 1926, pp. 1436–1443.

Suppose, however, that instead of more than a thousand measurements we had only a very small number, as is so often the case in engineering work. Our estimation of the variability of the distribution function representing the state of control upon the basis of the information given by the sample would necessarily be quite different from that ordinarily used by engineers, see Fig. 10. This is true even though to begin with we make the same kind of assumption as engineers have been

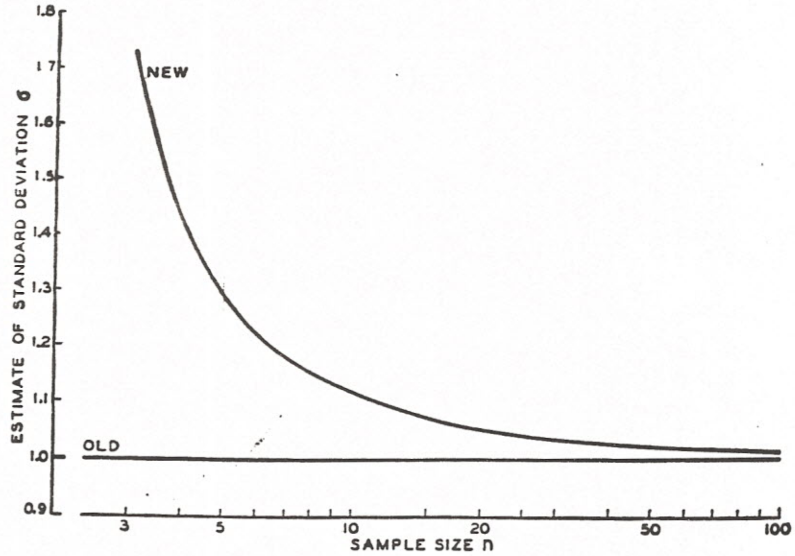

FIG. 10.—CORRECTION FACTORS MADE POSSIBLE BY MODERN STATISTICAL THEORY ARE OFTEN LARGE—TYPICAL ILLUSTRATION.

accustomed to make in the past. This we may take as a typical example of the fact that the production engineer finds it to his advantage to keep abreast of the developments in statistical theory. Here we use *new* in the sense that much of the modern statistical theory is new to most engineers.

5. *Conclusion*

Based upon evidence such as already presented, it appears feasible to set up criteria by which to determine when assignable causes of variation in quality have been eliminated so that the product may then be considered to be controlled within limits. This state of control appears to be, in general, a kind of limit to which we may expect to go economically in finding and removing causes of variability without changing a major portion of the manufacturing process as, for example, would be involved in the substitution of new materials or designs.