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MODEL REDUCTION IN THE PHYSICAL DOMAIN

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ABSTRACT

This paper is concerned with obtaining physical based low order approximations of high order models of linear physical systems with uniform parameters. Low order models result in several advantages including the reduction of computational complexity and improved understanding of the original system structure. Although different methods have been suggested for obtaining suitable low order approximations, these approaches do not reflect the relation between the mathematical model and the components of the physical system. Consequently, these procedures do not indicate which of the physical subsystems should be retained or eliminated in a possible reduced order model. In a previous work, a physically based model reduction procedure that is based on identifying subsystems of a physical system was presented. This paper is an enhancement of that physical domain model reduction technique. In this enhancement, the possibility of physical systems' having uniform parameters, or having similar bond graph loop gains is considered. In such cases the information from eigenvalue / eigenvector analysis has to be utilized in order to identify the components that are irrelevant to a specified mode of the physical system. The physical domain model reduction procedure and this enhancement is applied to a physical system.

NOMENCLATURE

$\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ system matrices
b damper coefficient (N sec/m)
C bond graph capacitance element
F force input (N)
F_{k_i} force in spring i (N)
G_{IC} $I-C$ loop gain
G_{IR} sum of loop gains of $I-R$ pairs
G_{RC} sum of loop gains of $R-C$ pairs
I bond graph inertial element
k spring coefficient (N/m)
m mass (kg)
$\mathbf{M}$ Right eigenvector matrix
R bond graph resistance element
$\mathbf{u}$ input vector
$\mathbf{v}$ right eigenvector
$\mathbf{w}$ left eigenvector
$\mathbf{W}$ Left eigenvector matrix
$\mathbf{x}$ state vector
$\mathbf{y}$ output vector
0 bond graph common effort junction
1 bond graph common flow junction
λ eigenvalue
ω frequency (rad/sec)
σ Hankel singular value
ζ_{i-j} local damping ratio of local loop between mass i and mass j

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INTRODUCTION

For complex dynamic systems, it is often useful to find a simplified model for purposes such as controller design, parameter optimization, design assessment under uncertainty, and to get better insight into the system behavior. Consequently, reduced order models have been studied in control system analysis for many years, and several methods have been suggested for determining low order approximations.

Model reduction techniques serve two important tasks in solving control-engineering problems. First, a model order reduction is often necessary to render control design problems to a manageable size when using modern control synthesis methods. Second, the controller reduction is useful to obtain simpler, hardware / software controllers that can be implemented more easily.

It is frequently desired to approximate the high order model by a reduced order model in such a way that the relevant dynamics is preserved in the low order model. Mathematically, this is usually done by the minimization of a suitable error norm. Most of the techniques in the literature take into account a criterion for the "goodness" of the reduced model. For example, the balancing approach (Moore, 1981) uses coordinate transformations to convert the system to a special balanced form from which a reduced model can be obtained. There exist several time and frequency domain methods, which generally provide good approximations. These include the component cost analysis for model reduction (Skelton and Yousuff, 1983) that uses a cost measure for eliminating modes, Padé approximations (Xiang, 1987) and continued fraction methods (Shieh and Gaudino, 1974) that employs the continued fraction expansion and inversion processes, approximate moment matching (Davidson and Walters, 1988) method that utilizes the elimination of some time moments, balance and truncate type of approaches (Safonov and Chiang, 1989) that exploit the balancing idea of (Moore, 1981), least squares model reduction (Lalonde et. al., 1992) method that uses the power of curve fitting, to name some of them.

In addition to these pure "numerical" procedures in the literature, a very useful type of reduced models is obtained by removing as many physical components as possible from the original model. This approach is known as model reduction in physical domain (Orbak, 1998; Orbak et. al., 2002). In this approach, the resultant models preserve the physical meanings of their structures and parameters, which are essential to analysis, synthesis and simulation. From analysis point of view, the information on physical structures and parameters provides better understanding of the effects of components' contribution to system dynamic behavior. From synthesis point of view, such reduced models can be used as the basis of much simpler designs that achieve the performances of more complicated systems. From simulation point of view, the resultant reduced model may save considerable computational cost while providing meaningful data.

The technique presented here is an extension to the physical domain model reduction method (Orbak et. al., 2002) and it can be used for both physical models and physical based controllers. In the technique, first the local loop gain calculations on a bond graph model is performed. Then, if the system has uniform parameters, or has similar loop gains, the information from eigenvalue / eigenvector analysis tool is used to identify components that are irrelevant to a given mode (eigenvalue) (Ye and Youcef-Toumi, 2000). After this step, the causality information on bond graphs helps to indicate the subsystems of a physical system. As a last step, the reduced order models can be selected by using the same procedure as in (Orbak et. al., 2002).

In the following sections, first the physical domain model reduction method will be briefly given. Then the extension mentioned above will be introduced. Both of these procedures will be applied to a physical system.

PHYSICAL DOMAIN MODEL REDUCTION

In physical domain model reduction procedures (Orbak et. al., 2002), the system structure information is obtained by carrying out the decomposition in the physical domain. This is performed directly on the system model using the bond graph models which describe the dynamic behavior of physical system by the connection of lumped and idealized elements based on the principle of energy conservation. Here, a brief summary of the decomposition and the model reduction procedure is included for completeness.

In the literature, the decomposition method of a system into its fast and slow dynamics counterparts is available (Sueur and Dauphin-Tanguy, 1991). But this has been done usually on systems with only resistance and capacitance elements (R and C elements) or on systems with only resistance and inertial elements (R and I elements). When a system has R, C and I elements at the same time, the procedure is to use the causal paths that are a series of bond variables connecting one specific variable to another according to the causality assignment and loop gains. It tries to determine the heavily damped and lightly damped subsystems' eigenvalues. In order to identify them, the improved procedure described in the next sections can be applied.

DECOMPOSITION PROCEDURE FOR HEAVILY DAMPED SUBSYSTEMS

As explained above, the procedure is based on the local loop gains of bond graphs (Sueur and Dauphin-Tanguy, 1991; Huang, 1997). Once one establishes a bond graph model of the system, the method can be carried out for the identification of heavily damped subsystems as follows (Huang, 1997; Orbak et. al., 2002);

1. Replace all the C elements by flow sources with zero value, identify the remaining $R-I$ pairs which are

directly casually related. Denote these $R-I$ elements and and the involved junctions as part of the subsystem $\mathcal{H}$.

2. Restore the C elements which are replaced in the previous step, identify the C elements which are directly casually related to the above I elements. If $\sqrt{G_{IC}} \gg G_{IR}$, replace the C elements by flow sources with zero value. Denote these flow sources as part of the subsystem $\mathcal{H}$. If $\sqrt{G_{IC}} \ll G_{IR}$, ignore the identified C elements.
3. Identify the I elements which become dependent due to the causalities imposed by the above sources. Denote these I elements as part of subsystem $\mathcal{H}$.
4. Replace all the I elements by effort sources with zero value, identify the remaining $R-C$ pairs which are directly casually related. Denote these $R-C$ elements and the involved junctions as part of the subsystem $\mathcal{H}$.
5. Restore the I elements which are replaced in the previous step, identify the I elements which are directly causally related to the above C elements. If $\sqrt{G_{IC}} \gg G_{RC}$, replace the I elements by effort sources with zero value. Denote these effort sources as part of the subsystem $\mathcal{H}$. If $\sqrt{G_{IC}} \ll G_{RC}$, ignore the identified I elements.
6. Identify the C elements which become dependent due to the causalities imposed by the above sources. Denote these C elements as part of subsystem $\mathcal{H}$.
7. Identify the resistances which are involved in heavily-damped local loops (loops with very large local damping ratios). Denote these R elements and the involved $I-C$ pairs, junctions as part of subsystem $\mathcal{H}$.
8. Identify the C elements which are not involved in the previous step, but are directly causally related to the above I elements. If $\sqrt{G_{IC}} \gg G_{IR}$, replace the C elements by flow sources with zero value. Denote these flow sources and the involved junctions as part of the subsystem $\mathcal{H}$. If $\sqrt{G_{IC}} \ll G_{IR}$, ignore the identified C elements.
9. Identify the I elements which are not involved in step 7, but are directly causally related to the above C elements. If $\sqrt{G_{IC}} \gg G_{RC}$, replace the I elements by effort sources with zero value. Denote these effort sources as part of subsystem $\mathcal{H}$. If $\sqrt{G_{IC}} \ll G_{RC}$, ignore the identified I elements.
10. Remove the elements which are not denoted as part of the subsystem $\mathcal{H}$. The remaining subsystem is the heavily-damped subsystem $\mathcal{H}$.

Using this procedure, the heavily damped portion of a large physical system can be identified. In some simple cases, all of the individual subsystems that are heavily damped can be identified. Similarly, the decomposition procedure for the identification of lightly damped subsystems is given in the next section.

DECOMPOSITION PROCEDURE FOR LIGHTLY DAMPED SUBSYSTEMS

In order to identify the lightly damped subsystems we can use the following procedure, which is much simpler than the first one:

1. Identify the $I-C$ pairs which are involved in lightly-damped local loops (loops with small local damping ratios), denote these $I-C$ elements as part of the subsystem $\mathcal{L}$.
2. Identify the R elements which are not involved in the previous step, but are directly causally related to the above I or C elements. If $\sqrt{G_{IC}} \ll G_{RI}$ or $\sqrt{G_{IC}} \ll G_{RC}$, replace the resistive R elements by flow sources with zero value and conductive R elements by effort sources with zero value. Denote these sources as part of the subsystem $\mathcal{L}$.
3. Identify the energy storage elements which become dependent due to the causalities imposed by the above sources. Denote these elements as part of the subsystem $\mathcal{L}$.
4. Remove the elements which are not denoted as part of the subsystem $\mathcal{L}$. The remaining subsystem is the lightly-damped subsystem $\mathcal{L}$.

The local damping ratios can be calculated by $\frac{G_{RC}}{2\sqrt{G_{IC}}}$ and $\frac{G_{RI}}{2\sqrt{G_{IC}}}$, which is equivalent of determining the damping ratio of a second order system. By using the above procedures one can decompose the system into subsystems, identify the modes and then proceed with the model reduction. Thus a physical domain model reduction procedure is given as follows:

1. Obtain the linear lumped parameter model of the system and then draw the bond graph model.
2. Obtain the subsystems of the model using the decomposition procedures.
3. Compute the residues of the full order model. According to the definition of residues, the largest residue has the most contribution to the dynamic behavior of the system.
4. Match the eigenvalues that must be retained according to step 1 together with step 2, and consequently determine subsystems to be retained and eliminated.

In the resulting reduced order model, there can be a DC gain discrepancy that can be corrected easily. The DC gain difference may occur in cases where some parameters are eliminated without compensating their effects on the system. This is related to the various sensitivities of the system with respect to each of the parameter (Burrows and Turkay, 1982). It should be noted that if the output were a flow that has a zero steady-state value for a stable system, the model reduction method would not have any DC gain error.

In the next section we will give an example of this model reduction method.

A SINGLE-INPUT SINGLE-OUTPUT PHYSICAL EXAMPLE

In this section, we will present the use of the decomposition procedures and model reduction for a single-input single-output (SISO) physical system.

Consider the system in Figure 1. The bond graph representation of this system is shown in Figure 2. As it can be seen from the bond graph, this system has 5 independent energy storage elements in integral causality, so this system is of order 5. For the simulation purposes the following values are chosen for the given parameters: $m_1 = m_2 = 1$ kg, $k_1 = k_3 = 1$ N/m, $k_2 = 15$ N/m, $b_1 = b_3 = 0.2$ Nsec/m, $b_2 = 1$ Nsec/m. To obtain the state-space equations, power variables (in this case $\dot{x}_1, \dot{x}_2, F_{k_1}, F_{k_2}, F_{k_3}$) were chosen. Here F_{k_i} represents the force on the i'th spring.

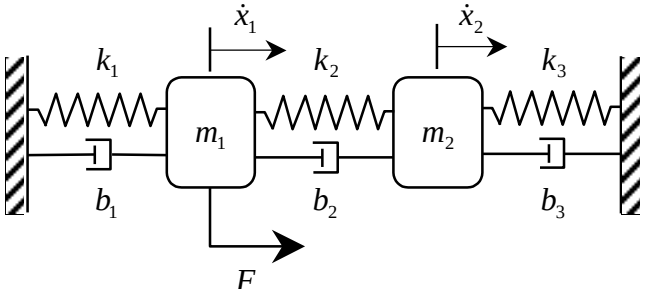


Figure 1. A 5th order physical system.

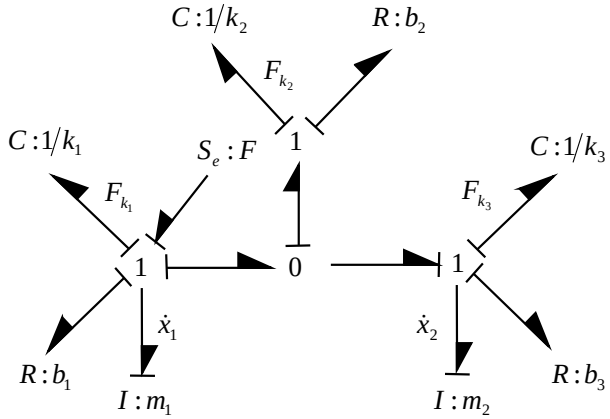


Figure 2. Bond graph representation of the 5th order system.

Once the equations of motion (or bond graph equations) are written, the following state-space representation is obtained:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{1}$$

where the $\mathbf{A}$ and $\mathbf{B}$ matrices are given by:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & k_1 & 0 \\ 0 & 0 & 0 & k_2 & -k_2 \\ 0 & 0 & 0 & 0 & k_3 \\ -\frac{1}{m_1} & -\frac{1}{m_1} & 0 & -\frac{b_1+b_2}{m_1} & \frac{b_2}{m_1} \\ 0 & \frac{1}{m_2} & -\frac{1}{m_2} & \frac{b_2}{m_2} & -\frac{b_2+b_3}{m_2} \end{bmatrix} \tag{2}$$

and

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{m_1} & 0 \end{bmatrix}^T \tag{3}$$

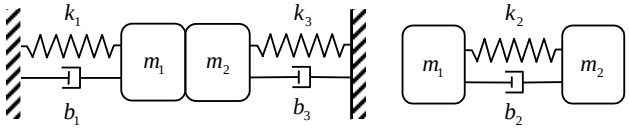
and if one is interested, let's say force on k_1, F_{k_1}, then the output equation will be:

$$y = \mathbf{C}\mathbf{x} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{x} \tag{4}$$

The model reduction procedure is applied as follows;

Step 1) **Modelling:** The bond graph model of the system is displayed in Figure 2.

Step 2) **Decomposition:** To perform the decomposition, the following local damping ratios and significant loop gains are calculated: $\frac{b_1}{2\sqrt{m_1 k_1}} = \frac{b_3}{2\sqrt{m_2 k_3}} = 0.1$, $\frac{b_2}{2\sqrt{m_1 k_2}} = \frac{b_2}{2\sqrt{m_2 k_2}} = 0.1291$, $\frac{k_2}{m_1} = \frac{k_2}{m_2} = 15$ rad/sec^2 and $\frac{k_1}{m_1} = \frac{k_3}{m_2} = 1$ rad/sec^2. The decomposition results into the two subsystems shown in Figure 3.



(a) Subsystem 1. (b) Subsystem 2.

Figure 3. Subsystems of the 5th order system.

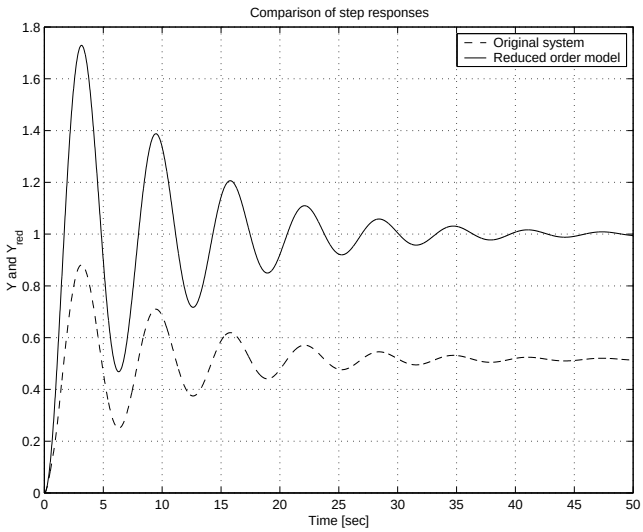
Step 3) **Relevant modes:** The calculated residues, their absolute values and the eigenvalues of the full order system are shown in Table 1. The largest residue indicates the mode that contributes most to the system behavior. Thus, the eigenvalues that correspond to the boldfaced residues should be retained in the reduced model of order 2.

Step 4) **Matching modes to subsystems:** Now we need to identify which subsystems need to be retained. One computes the eigenvalues of subsystem 1 as $\lambda_{1,2} = -0.1000 \pm 0.9950i$, and the eigenvalues of subsystem 2 as $\lambda_{3,4} = -1 \pm 5.3852i$, $\lambda_5 = 0$ which are shown in Table 1. Then we conclude that for a second order reduced model, subsystem 1 should be retained and subsystem 2 should be eliminated. This means that subsystem 1 will be the reduced 2nd order model. The time and

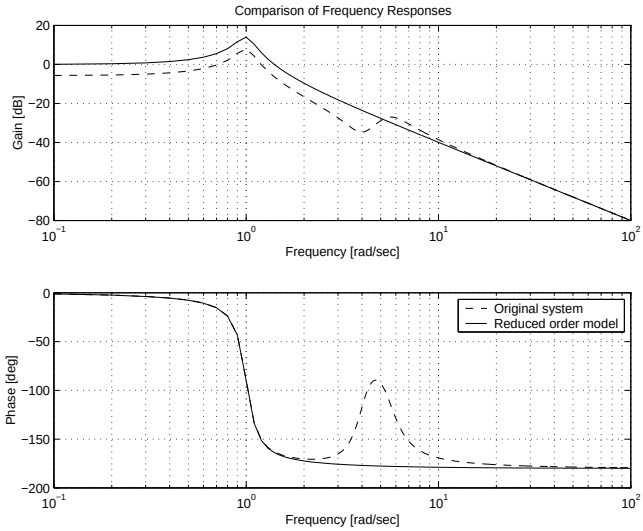
Table 1. Table for the residues and eigenvalues of the 5th order system.

Residue	**Absolute value**	**Eigenvalue**
$-0.0000 \mp 0.0458i$	0.0458	$-1.1000 \pm 5.4580i$
$-0.0000 \mp 0.2513i$	**0.2513**	$-0.1000 \pm 0.9950i$
-0.0000	0.0000	0.0000

frequency domain responses of the full order and reduced order models are compared in Figure 4. These plots indicate that the patterns are in a very good agreement except with a discrepancy of DC gain error, which can be eliminated by a constant calibration factor.



(a) Comparison of step responses.



(b) Comparison of Bode plots.

Figure 4. Comparison of full order and reduced order model using subsystems.

The frequency response plot shown in Figure 4(b) clearly indicates that the mode at $\omega = 5.458$ rad/sec is eliminated from the two-mode full order model. Hence, the frequency response plot justifies the result of our method.

Now, let's return to the question of uniform parameters[1]. Although this is a very special case, it enables us to see that in certain cases, depending on the parameter values of the elements in question, the local loop gains (and local damping ratios) that are calculated for the above method may produce same numerical result. In this case it may produce a problem to separate the system into smaller subsystems, i.e. we may separate the system into large subsystems, but cannot further distinguish different modes.

In such cases a method that depends on eigenvalue / eigenvector decomposition developed in (Ye and Youcef-Toumi, 2000) can be utilized. In what follows, we will first briefly outline the relevant tools and theorems of identifying irrelevant parameters to a given mode (eigenvalue) and then show the way to use the result for the physical domain model reduction methods.

STATE SPACE REPRESENTATION OF AN LTI SYSTEM

In this section the theorems and a procedure for the identification of the components that are irrelevant to a given eigenvalue of an LTI causal system is given (Ye and Youcef-Toumi, 2000). In our context irrelevancy means that changing the component's parameter value does not change the eigenvalue numerically. Physically, this indicates that the irrelevant component can be removed from the system without affecting the numerical value of the given eigenvalue. For this section, the generalized momenta / generalized displacements associated with independent inertances / capacitances are chosen as state variables.

Here, first, we will give a brief review of an existing procedure for the formulation of the state space equations of LTI systems (Rosenberg, 1971):

An LTI system can be characterized with several matrices that define the structure of the system. The parameters of the components can be described by two matrices, one for independent energy storage elements and one for dissipation elements. The energy storage elements can be represented by the matrix $\mathbf{S}$, defined as

$$\mathbf{z} = \mathbf{S}\mathbf{x} \tag{5}$$

where x_i is the generalized momentum / displacement associated with the i'th independent energy storage element (state) and z_i is the flow / effort as the causal output of that element. For an LTI system with all of

[1] A system has uniform parameters if all the inertance elements, capacitance elements and dissipation elements in the system have the same numeric parameter values respectively.

the independent energy storage elements (a total number of n) are one port, $\mathbf{S}$ is a diagonal matrix of the form $\text{diag}[s_1, s_2, \ldots, s_n]$, with s_i's as the parameters of the energy storage elements. From a computation point of view, if the i'th independent energy storage element is a capacitance (or an inertance), then $s_i = \frac{1}{C_i}$ (or respectively $s_i = \frac{1}{I_i}$).

On the other hand, the dissipation elements can be represented by the matrix $\mathbf{L}$, which contains the parameter values as follows: $\mathbf{d}_{out} = \mathbf{L}\mathbf{d}_{in}$, where d_{in_j} and d_{out_j} stand for the causal input and causal output of the j'th dissipation element, respectively. Again, for an LTI system with m one port dissipation elements, $\mathbf{L}$ is diagonal and of the form $\text{diag}[l_1, l_2, \ldots, l_m]$. From a computation point of view, when the j'th dissipation component has a flow (or an effort) as the causal input and an effort (respectively a flow) as the causal output, then $l_j = R_j$ (respectively $l_j = \frac{1}{R_j}$).

Then, the structure of a system is described by (Rosenberg, 1971):

$$\dot{\mathbf{x}} = \mathbf{J}_{SS}\mathbf{z} + \mathbf{J}_{SL}\mathbf{d}_{out} + \mathbf{J}_{SU}\mathbf{u} \quad (6)$$

$$\mathbf{d}_{in} = \mathbf{J}_{LS}\mathbf{z} + \mathbf{J}_{LL}\mathbf{d}_{out} + \mathbf{J}_{LU}\mathbf{u} \quad (7)$$

where,

$\mathbf{J}_{SS}$ describes the connections among the outputs of the energy storage elements and the inputs of the energy storage elements.
$\mathbf{J}_{SL}$ describes the connections among the outputs of the dissipation elements and the inputs of the energy storage elements.
$\mathbf{J}_{LS}$ describes the connections among the outputs of the energy storage elements and the inputs of the dissipation elements.
$\mathbf{J}_{LL}$ describes the connections among the outputs of the dissipation elements and the inputs of the dissipation elements.
$\mathbf{J}_{SU}$ describes the connections among the inputs from the sources $\mathbf{u}$ and the inputs of the energy storage elements.
And $\mathbf{J}_{LU}$ describes the connections among the inputs from the sources $\mathbf{u}$ and the inputs of the dissipation elements.

As a result, the system's state space equation is given by $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$, where

$$\mathbf{A} = \left[\mathbf{J}_{SS} + \mathbf{J}_{SL}\mathbf{L}(\mathbf{I} - \mathbf{J}_{LL}\mathbf{L})^{-1}\mathbf{J}_{LS}\right]\mathbf{S} \equiv \mathbf{J}\mathbf{S} \quad (8)$$

$$\mathbf{B} = \mathbf{J}_{SU} + \mathbf{J}_{SL}\mathbf{L}(\mathbf{I} - \mathbf{J}_{LL}\mathbf{L})^{-1}\mathbf{J}_{LU} \quad (9)$$

In the next section, we will briefly give the theorems and the procedure given in (Ye and Youcef-Toumi, 2000).

THEOREMS AND THE PROCEDURE FOR IDENTIFYING COMPONENTS IRRELEVANT TO AN EIGENVALUE

This section presents the main theorems and the corresponding procedure for identifying the irrelevant components of a given eigenvalue. The proofs of these theorems and detailed illustrations-applications for the procedure can be found in (Ye and Youcef-Toumi, 2000).

Theorem 1. *The independent energy storage elements that correspond to the zero components of the right eigenvector corresponding to the i'th mode are irrelevant to the i'th eigenvalue.*

Theorem 2. *The energy storage elements that correspond to the zero components in the left eigenvector corresponding to the i'th mode are irrelevant to the i'th eigenvalue.*

Theorem 3. *A resistance is irrelevant to the i'th eigenvalue λ_i if its causal input is a linear combination of the outputs of the energy storage elements irrelevant to λ_i.*

Keeping in mind these theorems, the following procedure (Ye and Youcef-Toumi, 2000) can be outlined for the identification of components that are irrelevant to a given eigenvalue, λ_i:

1. The first step is to obtain the right and left eigenvectors of $\mathbf{A}$ associated with the given λ_i.
2. Then, identify the energy storage elements associated with the zero elements in the right and left eigenvectors using the matrix $\mathbf{S}$.
3. As a third step, identify the dissipation elements whose inputs are linear combinations of the outputs of the energy storage elements identified in step 2. The identification of such dissipation elements can be performed as follows:
   (a) For each of the resistance elements, follow the causal path initiated at its output, till every branch of the causal path has reached an energy storage element or a source.
   (b) If every energy storage element reached by the branches of the causal path is associated with a zero element of the right and left eigenvector, then the input to the resistance is a linear combination of these energy storage elements.
4. Collect the elements identified in step 2 and 3. These elements are irrelevant to the given eigenvalue λ_i.

With this outline, we have completed our discussion. Now let's see how we can enhance the physical domain model reduction procedure:

As discussed above, when a system has uniform parameters or has numerically same loop gains, the usual physical domain model reduction procedure may fail to identify all of the modes of the system. In such cases the following sub-procedure improves the results:

1. First, the component that is irrelevant to a given mode is identified. (This given mode is the mode that the decomposition procedure identifies as a candidate for a subsystem.)
2. Then the causal paths with this component and the rest of the system is examined on the bond graph.
3. The irrelevant component and the components that have a causal relation to it is put in a separate subsystem.

Once the subsystems are identified the rest of the physical domain model reduction technique is applied without any change, i.e. the residue information is used to select the physical reduced order model. In the next section, this enhancement will be applied to the same physical system we used before.

A PHYSICAL EXAMPLE

Now let's suppose that we are dealing with the same system in Figure 1. But this time let's assume that the system has uniform parameters, i.e. $m_1 = m_2 = 1$ kg, $k_1 = k_2 = k_3 = 2$ N/m, $b_1 = b_2 = b_3 = 1$ Nsec/m. As one can see, the decomposition procedure in the previous sections does not recognize the two modes of the system in a clear manner. This is because the local damping ratios and loop gains are numerically exactly the same, and sum of loop gains are not much farther than each other. Still the procedure seems to indicate the two modes of the system in Figure 3. So, let's apply the sub-procedure outlined in the previous section to see its effectiveness.

For this system, the following system matrices can be constructed:

$$\mathbf{S} = \begin{bmatrix} \frac{1}{m_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 & 0 \\ 0 & 0 & k_1 & 0 & 0 \\ 0 & 0 & 0 & k_2 & 0 \\ 0 & 0 & 0 & 0 & k_3 \end{bmatrix} \tag{10}$$

$$\mathbf{J}_{SS} = \begin{bmatrix} 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

$$\mathbf{L} = \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix} \tag{12}$$

$$\mathbf{J}_{SL} = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{13}$$

$$\mathbf{J}_{LS} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

$$\mathbf{J}_{LL} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{15}$$

With these matrices the $\mathbf{A}$ matrix of the system is obtained as:

$$\mathbf{A} = \begin{bmatrix} -\frac{b_1+b_2}{m_1} & \frac{b_2}{m_2} & -k_1 & -k_2 & 0 \\ \frac{b_2}{m_1} & -\frac{b_2+b_3}{m_2} & 0 & k_2 & -k_3 \\ \frac{1}{m_1} & 0 & 0 & 0 & 0 \\ \frac{1}{m_1} & -\frac{1}{m_2} & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

As a result, the right and left eigenvector matrices for this $\mathbf{A}$ matrix are computed as:

$$\mathbf{M} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & \mathbf{v}_4 & \mathbf{v}_5 \end{bmatrix} \tag{17}$$

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 & \mathbf{w}_4 & \mathbf{w}_5 \end{bmatrix} \tag{18}$$

where

$$\mathbf{v}_1 = \begin{bmatrix} -0.4193 + 0.3969i \\ 0.4193 - 0.3969i \\ 0.2329 + 0.0361i \\ 0.4658 + 0.0722i \\ -0.2329 - 0.0361i \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} -0.4193 - 0.3969i \\ 0.4193 + 0.3969i \\ 0.2329 - 0.0361i \\ 0.4658 - 0.0722i \\ -0.2329 + 0.0361i \end{bmatrix}$$

$$\mathbf{v}_3 = \begin{bmatrix} 0.3906 + 0.4252i \\ 0.3906 + 0.4252i \\ 0.1836 - 0.3646i \\ 0 \\ 0.1836 - 0.3646i \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 0.3906 - 0.4252i \\ 0.3906 - 0.4252i \\ 0.1836 + 0.3646i \\ 0 \\ 0.1836 + 0.3646i \end{bmatrix}$$

$$\mathbf{v}_5 = \begin{bmatrix} 0 \\ 0 \\ -0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix}$$

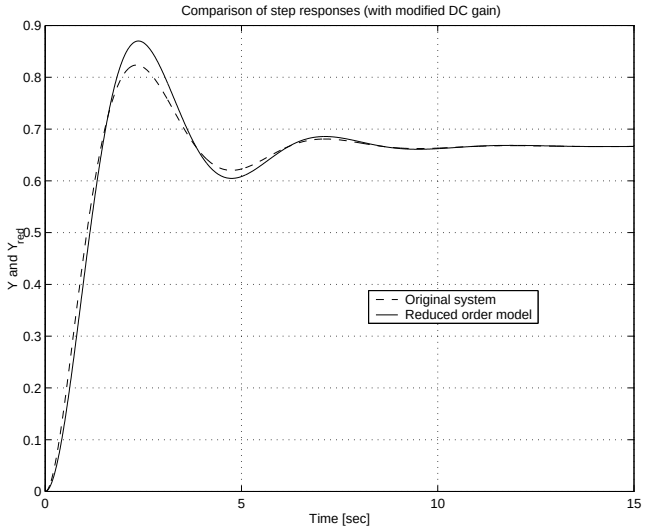
and

$$\mathbf{w}_1 = \begin{bmatrix} -0.0838 + 0.5413i \\ 0.0838 - 0.5413i \\ 0.3075 + 0.3248i \\ 0.6149 + 0.6495i \\ -0.3075 - 0.3248i \end{bmatrix}, \mathbf{w}_2 = \begin{bmatrix} -0.0838 - 0.5413i \\ 0.0838 + 0.5413i \\ 0.3075 - 0.3248i \\ 0.6149 - 0.6495i \\ -0.3075 + 0.3248i \end{bmatrix}$$

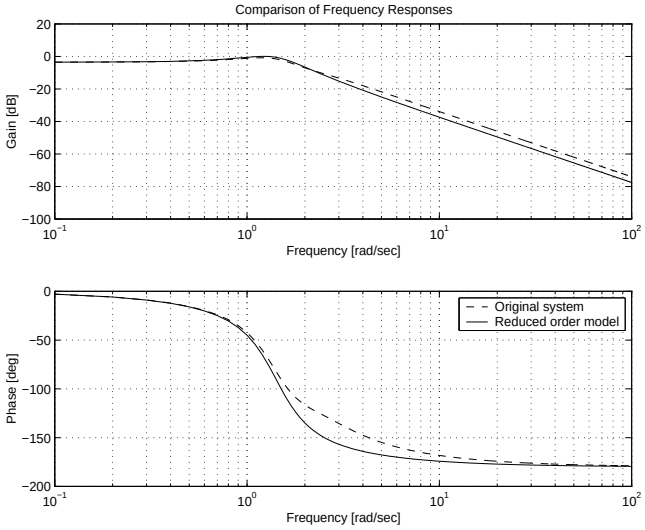
$$\mathbf{w}_3 = \begin{bmatrix} 0.4135 + 0.2082i \\ 0.4135 + 0.2082i \\ 0.4821 - 0.4429i \\ 0 \\ 0.4821 - 0.4429i \end{bmatrix}, \mathbf{w}_4 = \begin{bmatrix} 0.4135 - 0.2082i \\ 0.4135 - 0.2082i \\ 0.4821 + 0.4429i \\ 0 \\ 0.4821 + 0.4429i \end{bmatrix}$$

$$\mathbf{w}_5 = \begin{bmatrix} 0 \\ 0 \\ -0.5774 \\ 0.5774 \\ 0.5774 \end{bmatrix}$$

According to the procedure on identifying irrelevant components, k_2 is the irrelevant component for the eigenvalue $-0.5000+1.3229i$, and $-0.5000-1.3229i$. Now the application of the sub-procedure outlined in the previous section immediately gives the same two modes of Figure 3. This verifies that the identified modes are correct. If the physical domain model reduction is worked out as usual, the simulated results are as shown in Figure 5[2]. As it can be seen from this figure, the results are acceptable and the reduced order model provides a very good approximation.



(a) Comparison of step responses.



(b) Comparison of Bode plots.

Figure 5. Comparison of full order and reduced order model with uniform parameters.

CONCLUSION

In this paper, an extension for a physical domain model reduction method is presented. As discussed earlier, current model reduction techniques use numerical approaches and the resulting reduced order models do not have physical connections to the original physical system. By introducing a straightforward physical system decomposition procedure, one can find subsystems of a physical model. These subsystems consequently are related to the modes of that system. Then, by looking at the residues and their absolute values one can determine the relevant subsystems, and accordingly conclude which subsystems to retain in a reduced order model. Although this is the case, if the system has uniform parameters, or have such parameter values that result in same numerical values for local loop gains and local damping ratios, it may be difficult to further decompose a system, or identify different modes. This time a procedure that is used to identify the irrelevant components for a given eigenvalue can be utilized to improve the results. Both the physical domain model reduction method and its extension is applied to a physical system.

[2]The simulation results are with modified DC-gain

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