An LMI Based Approach for Velocity Feedback Vibration Controller Synthesis

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Abstract

This paper presents a linear matrix inequality (LMI) based approach combined with the cone complementarity algorithm for the synthesis of H_∞ velocity feedback full/reduced order dynamic and static vibration controllers. The resulting controllers are considered to be decentralized and positive real. In order to form the necessary constraints, linear matrix inequalities (LMIs) are used. Several examples are presented to demonstrate the results of the approach including the actual realization of the controller using dissipative elements.

1. Introduction

Vibration control is an important issue for structures subjected to different types of loading conditions, i.e. disturbance forces that induce severe vibration. The main characteristic describing the intensity of vibration is the peak vibration amplitude which have considerably important effects on the performance and safety of the system. The peak values of the vibration can be eliminated using various control approaches [1], [2], [3]. The velocity feedback control [8] is one of such methods. In this method, velocity values of the system are utilized to create the control action that can either be in static or dynamic state. This paper considers both cases of velocity feedback.

The decision of selecting the controller parameters depends on the performance requirements of both the controller and the uncontrolled system. The starting point for the formulation of desired system specifications is the bounded real lemma for the closed loop system. There are several constraints on the controller transfer function including being decentralized and positive real. The decentralized controller structure is in a diagonal or block diagonal form, thus the input/output pairing can be established. On the other hand, the positive realness is the key criteria in order to design passive controllers [4] made up of masses, springs and dampers.

In order to develop a useful synthesis method of such multiobjective controllers, linear matrix inequalities (LMIs) are a useful tool providing an algebraic representation of many control specifications [4]. LMI based methods enjoy efficient polynomial time convex optimization algorithms to solve LMIs. LMIs are also used to represent the constraints on system and controller performances [5], [6], [7]. This paper considers a combined LMI-cone complementarity algorithm in order to calculate both the dynamic and static velocity feedback controller parameters.

2. H_∞ problem and formulation

In this section, a n_p^{th} order linear time-invariant generalized plant P that contains what is usually called the vibrating plant in a vibration control problem will be considered. The generalized plant P also includes all frequency-dependent weighting functions. The plant transfer function can be written in state-space form as follows:

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix} \quad (1)$$

where the matrices are arranged in the form;

$$\begin{aligned} B_2 &= \begin{bmatrix} B_{21} & B_{22} & \cdots & B_{2N} \end{bmatrix} \\ D_{12} &= \begin{bmatrix} D_{121} & D_{122} & \cdots & D_{12N} \end{bmatrix} \\ C_2 &= \begin{bmatrix} C_{21} & C_{22} & \cdots & C_{2N} \end{bmatrix}^T \\ D_{21} &= \begin{bmatrix} D_{211} & D_{212} & \cdots & D_{21N} \end{bmatrix}^T \\ D_{22} &= \begin{bmatrix} D_{221} & D_{222} & \cdots & D_{22N} \end{bmatrix} \end{aligned}$$

and $i = 1, .., N$, total number of controller forces acting on the plant.

The ultimate aim is to minimize the vibration amplitude vector, $z(t) \in R^{n_z}$. $y_i(t) \in R^{n_y}$ and $u_i(t) \in R^{n_u}$ are the ith observation vector representing the measured variables, in this case velocities, and corresponding ith control input vector, respectively. $x(t) \in R^{n_p}$ is the state vector of the system. The disturbance vector $w(t) \in R^{n_w}$ contains all external inputs, including disturbances, sensor noise, and commands. The matrices $A, B_1, B_2, C_{11}, D_{11}, D_{12}, C_{21}, D_{21}, D_{22}$ are constant and compatible in dimension with corresponding vectors. The standard assumptions for the system are used [10]:

A.1 $D_{11} = 0, D_{22} = 0$.

A.2 (A, B_1) is stabilizable and (C_1, A) is detectable.

A.3 (A, B_2) is stabilizable and (C_2, A) is detectable for existence of a stabilizing K.

A.4 In order to ensure a proper and realizable controller: $RankD_{12} = n_u$, $RankD_{21} = n_y$.

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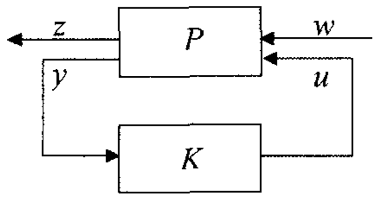

Figure 1. Generalized plant-controller configuration

A.5 $D_{12}^T \left[\begin{array}{cc} C_1 & D_{12} \end{array}\right] = \left[\begin{array}{cc} 0 & I \end{array}\right]$. It means that $C_1 x$ and $D_{12}u$ are orthogonal so that the penalty on $z = C_1 x + D_{12}u$ includes a nonsingular penalty on the control u.

A.6 $\left[\begin{array}{c} B_1 \\ D_{21} \end{array}\right] D_{21}^T = \left[\begin{array}{c} 0 \\ I \end{array}\right]$. It is dual of A.5 and concerns about how the exogenous signal w enters P: w includes both plant disturbance and sensor noise, these are orthogonal, and the sensor noise weighting is nonsingular.

A.7 $Rank \left[\begin{array}{cc} A - j\omega I & B_2 \\ C_1 & D_{12} \end{array}\right] = n_p + n_u$ and $Rank \left[\begin{array}{cc} A - j\omega I & B_1 \\ C_2 & D_{21} \end{array}\right] = n_p + n_y \; \forall \omega \in R$. To ensure that the optimal controller does not try to cancel poles or zeros on the imaginary axis which would result in closed-loop instability.

A.8 The controller is assumed to be collocated;

$$C_2 = B_2^T$$

The generic equations of motion for linear time-invariant dynamic controllers of fixed order n_c are given as:

$$\begin{array}{rcl} \dot{x}_c(t) & = & A_c x_c(t) + B_c y(t) \\ u(t) & = & C_c x_c(t) + D_c y(t) \end{array} \tag{2}$$

arranging in the matrix form one obtains:

$$\left[\begin{array}{c} \dot{x}_c \\ u \end{array}\right] = \left[\begin{array}{cc} A_c & B_c \\ C_c & D_c \end{array}\right] \left[\begin{array}{c} x_c \\ y \end{array}\right] \tag{3}$$

where $x_c \in R^{n_c}$ is the controller state.

On the other hand, the controller transfer function matrix is formed as:

$$K = \left[\begin{array}{cc} A_c & B_c \\ C_c & D_c \end{array}\right] \tag{4}$$

Now, for the static case; the equations of motion for linear time-invariant velocity feedback static controllers K are reduced to

$$u(t) = Ky(t) \tag{5}$$

with the controller $K = \bar{K}F$ where

$$\bar{K} = diag(k_{ii})$$

and F, a structure matrix consisting of ones, zeros and minus ones.

When a linear controller with transfer function $K(s)$ is inserted from y to u, the closed loop transfer function from w to z can be constructed as seen in Figure 1 [10]. If the open-loop system is augmented with the states corresponding to the controller, the following augmented system can be obtained:

$$\left[\begin{array}{c} \dot{x} \\ \dot{x}_c \\ \hline z \\ \hline x_c \\ y \end{array}\right] = \left[\begin{array}{cc|c|cc} A & 0 & B_1 & 0 & B_2 \\ 0 & 0 & 0 & I_{n_c} & 0 \\ \hline C_1 & 0 & D_{11} & 0 & D_{12} \\ \hline 0 & I_{n_c} & 0 & 0 & 0 \\ C_0 & 0 & D_{29} & 0 & D_{21} \end{array}\right] \left[\begin{array}{c} x \\ x_c \\ \hline w \\ \hline \dot{x}_c \\ u \end{array}\right] \tag{6}$$

equivalently,

$$\left[\begin{array}{c} \dot{\tilde{x}} \\ z \\ \tilde{y} \end{array}\right] = \left[\begin{array}{ccc} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & \tilde{D}_{52} \end{array}\right] \left[\begin{array}{c} \tilde{x} \\ w \\ \tilde{u} \end{array}\right]$$

where

$$\tilde{x} = \left[\begin{array}{c} x \\ x_c \end{array}\right], \tilde{y} = \left[\begin{array}{c} x_c \\ y \end{array}\right], \tilde{u} = \left[\begin{array}{c} \dot{x}_c \\ u \end{array}\right]$$

with

$$\tilde{u} = K\tilde{y}$$

$$\left[\begin{array}{c} \dot{\tilde{x}} \\ z \end{array}\right] = \left[\begin{array}{cc} \bar{A} & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_{11} \end{array}\right] \left[\begin{array}{c} \tilde{x} \\ w \end{array}\right].$$

The closed-loop system matrix can be written as an affine function of the controller matrix as follows:

$$\left[\begin{array}{cc} \bar{A} & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_{11} \end{array}\right] = \left[\begin{array}{cc} \tilde{A} & \tilde{B}_1 \\ \tilde{C}_1 & \tilde{D}_{11} \end{array}\right] + \left[\begin{array}{c} \tilde{B}_2 \\ \tilde{D}_{12} \end{array}\right] \breve{K} \left[\begin{array}{cc} \tilde{C}_2 & \tilde{D}_{21} \end{array}\right] \tag{7}$$

where

$$\breve{K} = \left[\begin{array}{cc} A_c + B_c(I - D_{22}D_c)^{-1}D_{22}C_c & B_c(I - D_{22}D_c)^{-1} \\ C_c(I - D_{22}D_c)^{-1} & D_c(I - D_{22}D_c)^{-1} \end{array}\right] \tag{8}$$

$\|T_{zw}\|_\infty$ denotes H_∞ norm of the closed-loop transfer function from w to z where $T_{zw} = \bar{D}_{11} + \bar{C}_1(sI - \bar{A})^{-1}\bar{B}_1$, i.e. its largest gain across frequency in the singular value norm. $\|T_{zw}\|_\infty < \gamma$ can be interpreted as a disturbance rejection performance, so the following lemma can be introduced:

Lemma 1: **Bounded Real Lemma** [5] Given a system of the form

$$\left[\begin{array}{c} \dot{\tilde{x}} \\ z \end{array}\right] = \left[\begin{array}{cc} \bar{A} & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_{11} \end{array}\right] \left[\begin{array}{c} \tilde{x} \\ w \end{array}\right] \tag{9}$$

then the following statements are equivalent:

i) $\|T_{zw}(s)\|_\infty < \gamma$

ii) there exists a positive definite matrix Q such that

$$\left[\begin{array}{ccc} \bar{A}^T Q + Q\bar{A} & Q\bar{B}_1 & \bar{C}_1^T \\ \bar{B}_1^T Q & -\gamma I & \bar{D}_{11}^T \\ \bar{C}_1 & \bar{D}_{11} & -\gamma I \end{array}\right] < 0. \tag{10}$$

Using the elimination Lemma [7] and following an algebraic procedure the following necessary and sufficient conditions for the H_∞ control problem can be obtained: There exists a controller that solves the fixed order H_∞ control problem if and only if there exist positive definite matrices X and Y such that

$$\left[\begin{array}{cc} \left[\begin{array}{c} B_2 \\ D_{12} \end{array}\right]^{\perp T} & \begin{array}{c} 0 \\ \end{array} \\ \begin{array}{c} 0 \end{array} & I \end{array}\right] \left[\begin{array}{ccc} AX + XA^T & XC_1^T & B_1 \\ C_1^T X & -\gamma I & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{array}\right] \left[\begin{array}{cc} \left[\begin{array}{c} B_2 \\ D_{12} \end{array}\right]^{\perp T} & \begin{array}{c} 0 \\ \end{array} \\ \begin{array}{c} 0 \end{array} & I \end{array}\right]^T < 0 \tag{11}$$

$$\begin{bmatrix} \begin{bmatrix} C_2^T \\ D_{21}^T \end{bmatrix}^{\perp T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A^TY + YA & YB_1 & C_1^T \\ B_1^TY & -\gamma I & D_{11}^T \\ C_1 & D_{12} & -\gamma I \end{bmatrix} \begin{bmatrix} \begin{bmatrix} C_2^T \\ D_{11}^T \end{bmatrix}^{\perp T} & 0 \\ 0 & I \end{bmatrix}^T < 0 \tag{12}$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \tag{13}$$

$$Rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \leq n_p + n_c \tag{14}$$

Note that, the rank constraint exists whenever the order of the controller is smaller than the order of the plant. For the static controller case, order of the controller is $n_c = 0$. The relation

$$Rank(I - XY) \leq n_c$$

can be written as

$$Rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix}$$

$$= Rank \begin{bmatrix} I & 0 \\ -X^{-1} & I \end{bmatrix} \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \begin{bmatrix} I & X^{-1} \\ 0 & I \end{bmatrix}.$$
$$= Rank \begin{bmatrix} X & 0 \\ 0 & Y - X^{-1} \end{bmatrix}.$$
$$\leq Rank(Y - X^{-1}) + Rank(X)$$

and it can be obtained that [5]

$Rank(X) = n_p$, $Rank(Y - X^{-1}) = Rank(Y_{12}Y_{22}^{-1}Y_{12}^T) \leq n_c$.

Then by introducing the notation

$$Q = \begin{bmatrix} Y & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, Q^{-1} = \begin{bmatrix} X & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \tag{15}$$

where $X, Y \in R^{n_p x n_p}$ and $X_{22}, Y_{22} \in R^{n_c x n_c}$ and inserting the expressions for the closed-loop matrices in the bounded real lemma condition, the following bilinear matrix inequality (BMI) formulation of the H_∞ control problem can be obtained:

Find a parameter matrix $Q > 0$ and a controller matrix $\hat{K}$ such that

$$\begin{bmatrix} (\tilde{A} + \tilde{B}_2\hat{K}\tilde{C}_1)^TQ + Q(\tilde{A} + \tilde{B}_2\hat{K}\tilde{C}_2) & Q(\tilde{B}_1 + \tilde{B}_2\hat{K}\tilde{D}_{21}) & (\tilde{C}_1 + \tilde{D}_{12}\hat{K}\tilde{C}_2)^T \\ (\tilde{B}_1 + \tilde{B}_2\hat{K}\tilde{D}_{21})^TQ & -\gamma I & (\tilde{D}_{11} + \tilde{D}_{12}\hat{K}\tilde{D}_{21})^T \\ (\tilde{C}_8 + \tilde{D}_{12}\hat{K}\tilde{C}_2) & (\tilde{D}_{11} + \tilde{D}_{12}\hat{K}\tilde{D}_{21}) & -\gamma I \end{bmatrix} < 0 \tag{16}$$

Equation (16) can be solved by standard LMI Toolbox of Matlab or any other LMI solvers.

3. Constraints on the controller

In this section, several constraints on the controller will be formulized via LMI's.

3.1. Decentralized controller

For a decentralized controller with N-controller force action on the plant; the matrices A_c, B_c, C_c, D_c; consist of N sub-matrices $\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i$; in the following form:

$$\begin{aligned} A_c &= diag([\hat{A}_1]_{\hat{n}_1 \times \hat{n}_1}, [\hat{A}_2]_{\hat{n}_2 \times \hat{n}_2}, \cdots, [\hat{A}_N]_{\hat{n}_N \times \hat{n}_N})_{n_c \times n_c} \\ B_c &= diag([\hat{B}_1]_{\hat{n}_1 \times 1}, [\hat{B}_2]_{\hat{n}_2 \times 1}, \cdots, [\hat{B}_N]_{\hat{n}_N \times 1})_{n_c \times N} \\ C_c &= diag([\hat{C}_1]_{1 \times \hat{n}_1}, [\hat{C}_2]_{1 \times \hat{n}_2}, \cdots, [\hat{C}_N]_{1 \times \hat{n}_N})_{N \times n_c} \\ D_c &= diag([\hat{D}_1]_{1 \times 1}, [\hat{D}_2]_{1 \times 1}, \cdots, [\hat{D}_N]_{1 \times 1})_{N \times N} \end{aligned} \tag{17}$$

with

$$n_c = \sum_{i=1}^{N} \hat{n}_i \tag{18}$$

3.2. Positive realness

The last lemma that needs to be introduced is the positive real lemma which will add the positive realness property to the obtained controller.

Lemma 2: **Positive Real Lemma** [5] The passivity property for positive realness of the controller is equivalent to the existence of any matrix $W = W^T > 0$ such that

$$\begin{bmatrix} A_c^TW + WA_c & C_c^T - WB_c \\ C_c - B_c^TW & -(D_c^T + D_c) \end{bmatrix} \leq 0. \tag{19}$$

For the static controller case; this reduces to

$$k_{ii} > 0 \tag{20}$$

It should be pointed out that, this condition automatically guarantees the stability of the closed loop system.

4. A combined LMI-cone complementarity algorithm

The following sets in the space of symmetric matrices should be defined before the introduction of the algorithm [10]:

$\mathrm{D} = \{Z \in \mathrm{S}^{2n} : Z = \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}, X, Y \in \mathrm{S}^n\}$,

$\mathrm{R}_k = \{Z \in \mathrm{S}^{2n} : rank(Z + J) \leq k\}$,

where $k = n_p + n_c$ and

$J = \begin{bmatrix} 0 & I_{n_p} \\ I_{n_p} & 0 \end{bmatrix} \in \mathrm{S}^{2n}$.

It should be noticed that, in addition to the convex LMI constraint sets $(11 - 13)$, when the non-convex constraint exists, the following theorem should be used to compute the orthogonal projection onto the non-convex constraint set [10].

Theorem 1: [7] Let $Z \in \mathrm{S}^{2n}$ and let $Z + J = U\Sigma V^T$ be the singular value decomposition of $Z + J$. The orthogonal projection, $Z^* = P_{\mathrm{R}_k}Z$ onto the set R_k is given by

$$Z^* = U\Sigma_kV^T - J$$

where Σ_k is the diagonal matrix obtained by replacing the smallest $n_p - n_c$ singular values in $Z + J$ by zero.

As a result, the following algorithm can be constructed in order to obtain a feasible controller that satisfies positive realness [10]:

Step 1: Find X, Y that satisfy the LMI constraints $(11-13)$ and minimize γ. If the problem is infeasible, stop. Otherwise, $\gamma_{min} = \gamma$ and set $X_0 = X$,$Y_0 = Y$ and $k = 1$. Using (15), solve (16) for $\hat{K}$ and go to step 7. If the solution is infeasible, go to step 2.

Step 2: Set $\gamma_k = \gamma_{k-1} + \epsilon$ with $0 < \epsilon < 10e-2$. Find X_k,Y_k that solve the corresponding cone complementarity problem [6]:

Step 3: minimize $\mathbf{Tr}(X_{k-1}Y_k + X_kY_{k-1})$ subject to LMI's $(11-13)$.

Step 4: If the objective $\mathbf{Tr}(X_{k-1}Y_k + X_kY_{k-1})$ has reached a stationary point, go to Step 5. Otherwise, set $k = k+1$ and go to Step 3.

Step 5: Denote the minimizing solutions by (X^*, Y^*); that is, the projection onto Γ_{convex} is written as $(X^*, Y^*) = P_{\Gamma_{convex}}(X_0, Y_0)$, construct Z. If there exist non-convex constraints apply Theorem (1) and compute Z^*.

Step 6: Take $Q = Z$ or Z^* and solve the controller $\hat{K}$ in (16). If the solution is infeasible, go to step 2.

Step 7: When the positive realness constraint exists on the controller, check (19). If the controller satisfies (19), stop, else go to step 2.

5. Numerical examples

In this section, case studies on controller synthesis will be given. As a model, the same two degree of freedom system of [9], [10] is used. The system has the equations of motion

$$M\ddot{x} + D\dot{x} + K = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

where

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0.02 & -0.01 \\ -0.01 & 0.01 \end{bmatrix}, K = \begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix}$$

Harmonic excitation force acts on the first degree of freedom externally. The aim is to minimize the ∞-norm of the second degree of freedom. Assuming that the controller is acting on the second degree of freedom, the following state-space matrices are obtained [10]:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -8 & 4 & -0.02 & 0.01 \\ 4 & -4 & 0.01 & -0.01 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T, C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

5.1. Positive real full/reduced order dynamic and static controller synthesis

A 4th order (full) and a 2nd order (reduced) dynamic controllers were synthesized in [10]. Their results are recited here for completeness of this paper. For the full order controller, $\|T_{zw}(s)\|_\infty < \gamma_{min} = 1.5572$ was obtained, and the minimized vibration amplitude of the second degree of freedom was found to be 0.4924. The frequency response with and without controller is given in Figure 2 [10].

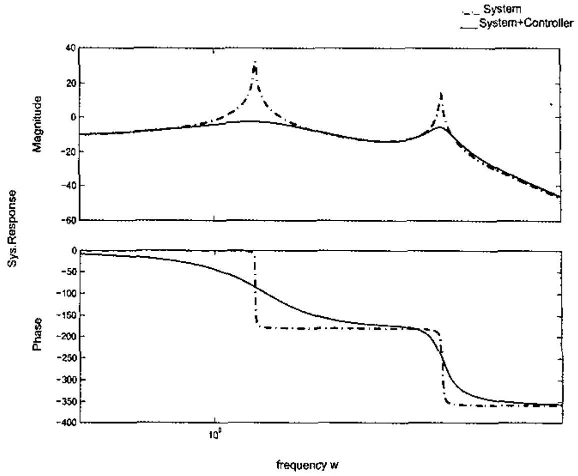

Figure 2. Results with positive real full order dynamic controller

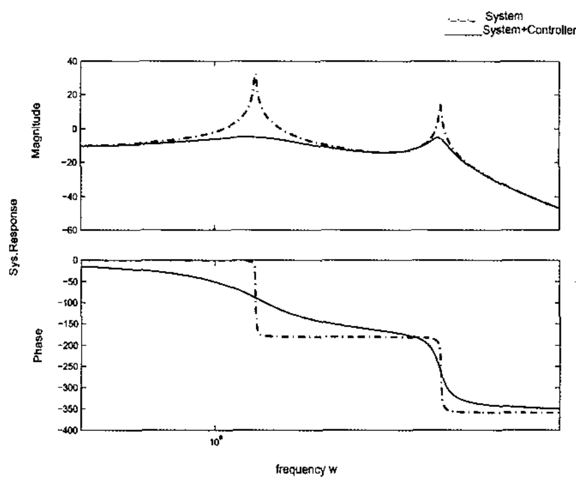

Figure 3. Results with positive real reduced order dynamic controller

On the other hand, for the reduced order controller, $\|T_{zw}(s)\|_\infty < \gamma_{min} = 1.6572$ was obtained. This time, the minimized vibration amplitude of the second degree of freedom was calculated to be 0.5650. It should be noted that this value is higher than the full order model as expected. The frequency response with and without controller is given in Figure 3 [10].

Now, as discussed in this paper, for a static controller synthesis of this problem, a velocity feedback controller is attached to the 2nd mass as seen in Figure 4. A reduced order controller is still desired. Once the optimization process is completed with the given constraints, the following

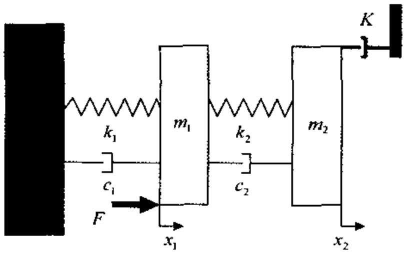

Figure 4. Controller on the 2nd mass

results are obtained: $\|T_{zw}(s)\|_\infty < \gamma_{min} = 1.8572$ and the minimized vibration amplitude of the second mass is 0.6646. The frequency response with and without controller is given in Figure 5. The designed controller is given as:

$$K = 0.7458$$

As a result, this controller can be interpreted as a damper with one side acting on the 2nd mass, whereas the other side is attached to a fixed frame.

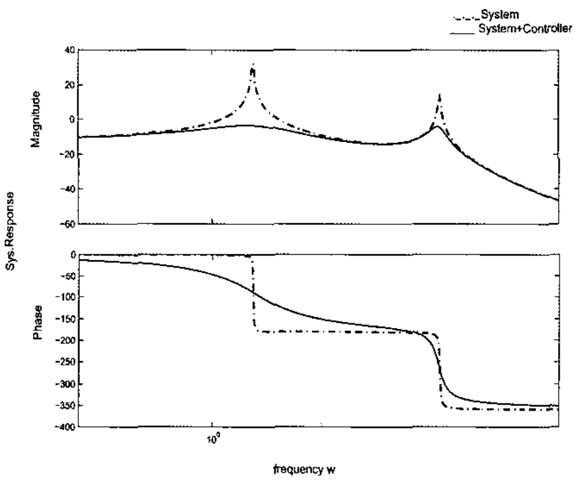

Figure 5. Results with positive real reduced order static controller

5.2. Decentralized controller synthesis

In this example, after giving the results of the 4th order decentralized dynamic controller, a decentralized static velocity feedback controller will be synthesized. In [10], the optimization process revealed that $\|T_{zw}(s)\|_\infty < \gamma_{min} = 2.0886$, and the minimized vibration amplitude of the second degree of freedom was 0.4924. The frequency response with and without controller is given in Figure 6 [10].

This time, a decentralized static velocity feedback controller will be synthesized. In order to achieve this, two controllers are attached to the two masses as seen in Figure 7.

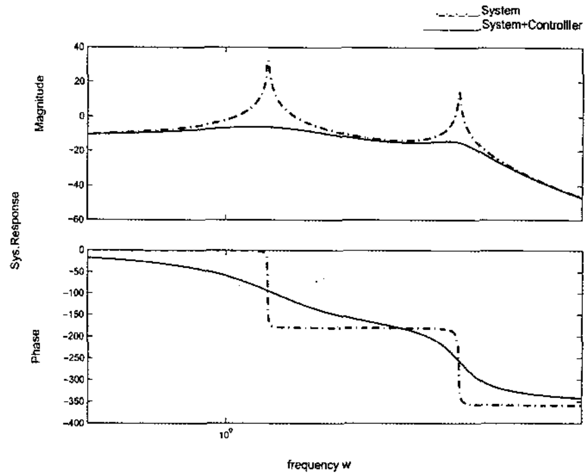

Figure 6. Results with positive real decentralized dynamic controller

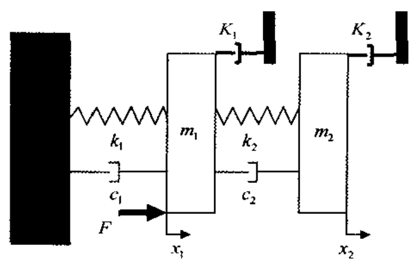

Figure 7. Two controllers on the masses

After the optimization process, the following results are obtained: $\|T_{zw}(s)\|_\infty < \gamma_{min} = 1.1886$. The minimized vibration amplitude of the second mass is 0.5489. The frequency response with and without controller is given in Figure 8. The designed controller is given as:

$$K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} = \begin{bmatrix} 0.6570 & 0 \\ 0 & 0.6354 \end{bmatrix}$$

These two controllers that has been designed can be interpreted as dampers with one sides acting on the masses, whereas the other sides are attached to a fixed frame.

5.3. Positive real static controller synthesis

For a last example, a static velocity feedback controller is inserted between the two masses, see Figure 9). The synthesis of this controller is as below.

After the optimization process, the following results are obtained: $\|T_{zw}(s)\|_\infty < \gamma_{min} = 3.9930$, and the minimized vibration amplitude of the second mass is 2.2455. The frequency response with and without controller is given in Figure 10. The designed controller is given as:

$$K = 1.7243$$

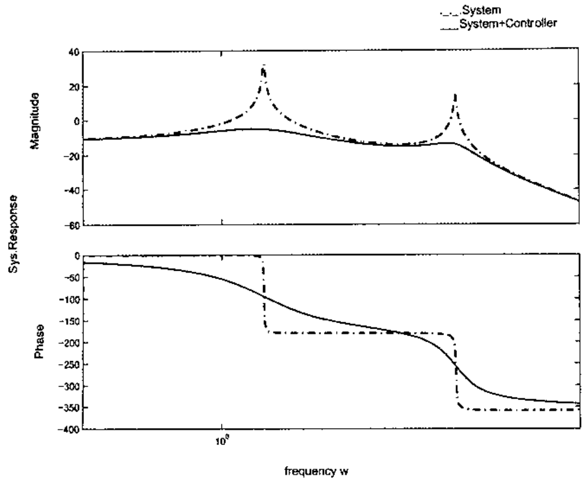

Figure 8. Results with positive real decentralized static controllers

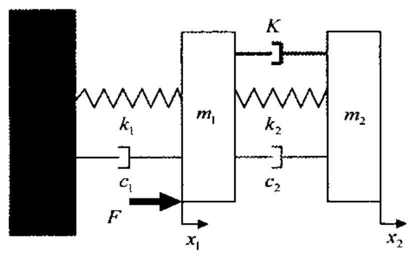

Figure 9. One controller between the masses

This computed controller can be, once more, interpreted as a damper inserted between the two masses.

6. Conclusion

In this paper, a solution method for the H_∞ control problem is presented using linear matrix inequality (LMI) approach. Several positive real velocity feedback controllers in full order, decentralized, and reduced order form are designed in both the dynamic and the static configuration. According to the simulations performed, it is concluded that the velocity feedback control system reduces the vibration response significantly. On the other hand, constraints on the system and controller transfer functions increase the H_∞ norm and give less effective results for the system performance.

One of the advantage of static controllers over dynamic ones is that they can be realized using dissipative elements, such as dampers. Future research will focus on controllers realized by the combination of masses, springs and dampers.

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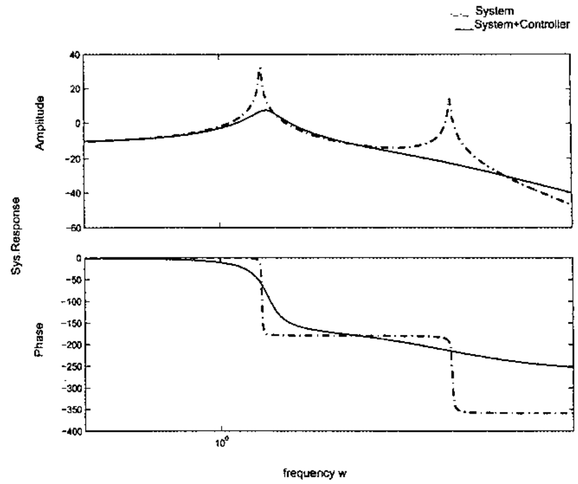

Figure 10. Results with positive real static controller

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