Modeling and Full State Feedback Adaptive Control of a Two Dimensional Linear Motor

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***Abstract*—In food processing and similar types of industries, packaging is performed in two steps, namely "primary packaging" and "secondary packaging". Generally the placement of the product on the pallets is done using custom designed mechanisms and conveyor systems. The desire for flexibility in such systems, produced robotic palletizing/collation systems. These systems usually include a cell incorporating a two-dimensional linear motor. This paper presents the development of an adaptive control for this type of linear motor. For this aim, first the modeling of the linear motor has been completed. After deciding on the best model, an adaptive controller has been suggested to improve the reliability of the system.**

***Index Terms*—Modeling, adaptive control, full state feedback control, two dimensional linear motors.**

I. Introduction

In a typical food processing industry, the "primary packaging" of the product may include boxes, plastic wrapping, aluminum or steel cans. Before food is shipped, it is collated into "secondary packaging", which may include boxes or trays. In general, the collation of the product into secondary packaging and its placement on the pallets is done via custom designed mechanisms and conveyor systems. The end result is an efficient, reasonably reliable system, which nevertheless is very limited in flexibility and requires periodic maintenance. Any change in the size of the pack, the pallet or the secondary packaging requires significant changes in the system hardware. This desire for flexibility was the motivation for a robotic palletizing/collation system.

Such a robotic palletizing system had been built at Massachusetts Institute of Technology (MIT) for research and development. It is a cell incorporating a two-dimensional linear motor. The motor moves on the horizontal plane underneath a ferromagnetic platen, from which it is separated by an air gap. An end effector is attached underneath the motor for picking up the packs from a conveyor belt and placing them on a pallet. This system had proven to be very fast and maintenance free.

However, the system without any controller does not have a reliable response. Proportional-integral-derivative (PID) control algorithms are unable to meet this basic requirement. After tuning the parameters of these controllers to optimize the tracking performance, it happens to be that tracking is optimized for some specific conditions only. The sources of this problem are mainly the nonlinearities introduced by the magnetic field distribution and eddy currents at high speeds. This work presents the development of an adaptive control for this linear motor. An adaptive controller has been suggested to improve the reliability of the system.

In the next section, first, the modeling of the two-dimensional linear motor is given and its response is compared to the response of the actual motor.

II. Modeling of the Linear Motor

A prerequisite for the development of a feedback control for the two dimensional linear electric motor is through the understanding of its dynamics. The standard tool for this understanding is the formulation of the mathematical model of the motor. In this context, "motor" and "plant" both refer to the physical entity or real system that is worked with.

In this section, the results obtained in [2] are briefly presented. This model analyzes the process of the electromechanical energy conversion in detail. Besides, the model takes into account the electromagnetic loss mechanisms in order to match the motor dynamics better than previous attempts [7].

As it is explained in section III, this model in its original version cannot be used in an adaptive control scheme. Consequently, many simplifications are applied in order to get a linear transfer function representing the plant of the system. With the help of simulation software the detailed model is tested and the results compared to measurements of the real plant in order to investigate its accuracy. Finally, a comparison between the detailed or fourth order model and the simplified models is presented so as to show the influence of each simplification.

A. Fourth Order Model

In the following subsections the equations describing the physical properties of the linear motor in the electrical, magnetic and mechanical domains are presented. This section is intended to be a brief presentation of previous results [2, 6, 7]. Thus, details related to the specific configuration of the system are not discussed. Instead, only the main concepts are involved.

i. Underlying Equations

This presentation of a detailed model of the two-dimensional linear electric motor includes a first simplification performed on the original development. This first simplification consists on taking into account the reluctance of the air gap only. In other words, the total reluctance of a magnetic circuit consisting of a motor core with legs and a base, an air gap, and a platen base is considered to be very close to the reluctance of the air gap only. The justification for this assumption is the fact that the relative permeability of the air is much smaller than that of the silicon steel used in cores and platen bases. Therefore, under same conditions of magnetic flux and transversal area, the reluctance of the air gap is much, much higher than that of the motor core and platen.

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Consequently, the reluctance of entire path is approximated by the reluctance of the air gap. Because the geometry of the air gap changes with the movement of the motor with respect to the platen, the reluctance happens to be a function of position only. This reluctance is modeled as a sinusoid of position with a period of one pitch. Obeying the x coordinate axis definition of Figure 1, the reluctances are represented by:

$$R_{iG} = R_{iG}(x) = R_{G0}\left(1 + k_G \sin\left(\frac{2\pi x}{p}\right)\right) \qquad i = 1,3 \qquad (1)$$

$$R_{iG} = R_{iG}(x) = R_{G0}\left(1 - k_G \sin\left(\frac{2\pi x}{p}\right)\right) \qquad i = 2,4 \qquad (2)$$

$$R_{iG} = R_{iG}(x) = R_{G0}\left(1 + k_G \cos\left(\frac{2\pi x}{p}\right)\right) \qquad i = 5,7 \qquad (3)$$

$$R_{iG} = R_{iG}(x) = R_{G0}\left(1 - k_G \cos\left(\frac{2\pi x}{p}\right)\right) \qquad i = 6,8 \qquad (4)$$

where

$$R_{G0} = \frac{392500}{H} \qquad (5)$$

$$k_G = 0.45 \qquad (6)$$

On the other hand, if it is assumed that the magnetic flux confines itself to the ferromagnetic material, that leakage and fringing can be neglected, and that the permanent magnet can be modeled as a source of magnetic flux incorporating a magnetomotive force M_{PM} and a reluctance R_{PM}, the expressions for the magnetizing currents on phases A and B are:

$$i_{mA} = -\frac{\left(R_1^2 + R_2^2\right)M_{PM}}{(R_1 + R_2)(R_1 + R_2 + 2R_{PM})n} + \frac{(R_1 + R_2)(2R_{12} + R_{PM})\phi_{23}}{(R_1 + R_2 + 2R_{PM})n} \qquad (7)$$

$$i_{mB} = -\frac{\left(R_5^2 + R_6^2\right)M_{PM}}{(R_5 + R_6)(R_5 + R_6 + 2R_{PM})n} + \frac{(R_5 + R_6)(2R_{56} + R_{PM})\phi_{67}}{(R_5 + R_6 + 2R_{PM})n} \qquad (8)$$

where:

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} \qquad R_{56} = \frac{R_5 R_6}{R_5 + R_6} \qquad (9)$$

and $n = 80$, is the number of turns of each winding of each phase and of each axis of the linear motor.

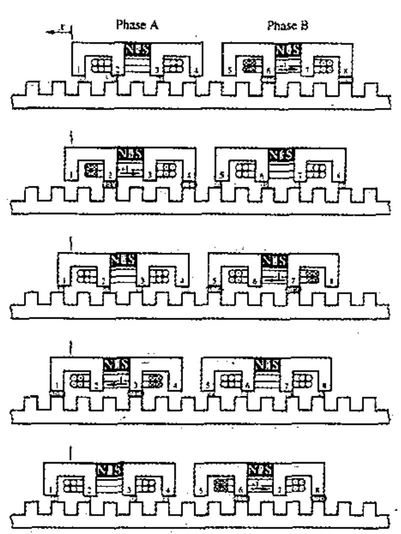

Fig. 1. A simple Sawyer linear motor, principle of operation.

Next, the magnetic flux through each one of the eighth pole faces of a set constituted of a phase A and a phase B like the one represented in Figure 1 is:

$$\phi_1 = \frac{R_2 M_{PM} - (R_2 + R_{PM})ni_{mA}}{(R_1 + R_2)(2R_{12} + R_{PM})} \qquad (10)$$

$$\phi_2 = \frac{R_1 M_{PM} + (R_1 + R_{PM})ni_{mA}}{(R_1 + R_2)(2R_{12} + R_{PM})} \qquad (11)$$

$$\phi_5 = \frac{R_6 M_{PM} - (R_6 + R_{PM})ni_{mA}}{(R_5 + R_6)(2R_{56} + R_{PM})} \qquad (12)$$

$$\phi_6 = \frac{R_5 M_{PM} + (R_5 + R_{PM})ni_{mA}}{(R_5 + R_6)(2R_{56} + R_{PM})} \qquad (13)$$

Because the reluctance of the magnetic path is approximated by taking into account the reluctance of the air gap only:

$$\phi_3 = -\phi_1 \qquad \phi_4 = -\phi_2 \qquad (14)$$

$$\phi_7 = -\phi_5 \qquad \phi_8 = -\phi_6 \qquad (15)$$

And the force generated by the electromechanical energy conversion is:

$$F = 8\pi k_G R_{G0}\left(\left(\phi_5^2 - \phi_6^2\right)\sin\left(\frac{2\pi x}{p}\right) + \left(\phi_2^2 - \phi_1^2\right)\cos\left(\frac{2\pi x}{p}\right)\right)p^{-1} \qquad (16)$$

The core and platen losses due to magnetic hysteresis and eddy currents in the mechanical domain are represented by a drag force. This agrees with the fact that the magnitude of eddy current losses is proportional to the time derivative of the magnetic flux, which is supposed to be directly proportional to the velocity.

Because the motor is gliding on an air cushion during operation, if the aerodynamic friction is neglected, this air bearing system can be considered lossless. Therefore, the net force acting on the total mass of motor and load must be the force generated minus the drag force. The drag force, as function of velocity only, was defined to be:

$$F_d = 1.295v^{0.6} + 52.213v \qquad (17)$$

Finally, if we define:

$$\phi_{23} = \phi_2 + \phi_3 \qquad (18)$$

$$\phi_{67} = \phi_6 + \phi_7 \qquad (19)$$

then the state space representation of the system can be expressed as:

$$\frac{d(\phi_{23})}{dt} = \frac{u_A}{n} + \frac{\left(R_1^2 + R_2^2\right)M_{PM} R_{DW}}{(R_1 + R_2)(R_1 + R_2 + 2R_{PM})n^2} - \frac{(R_1 + R_2)(2R_{12} + R_{PM})R_{DW}\phi_{23}}{(R_1 + R_2 + 2R_{PM})n^2}$$

$$\frac{d(\phi_{67})}{dt} = \frac{u_B}{n} + \frac{\left(R_5^2 + R_6^2\right)M_{PM} R_{DW}}{(R_5 + R_6)(R_5 + R_6 + 2R_{PM})n^2} - \frac{(R_5 + R_6)(2R_{56} + R_{PM})R_{DW}\phi_{67}}{(R_5 + R_6 + 2R_{PM})n^2}$$

$$\frac{dv}{dt} = \frac{F - F_d}{m}$$

$$\frac{dx}{dt} = v$$

where:

u_A = Voltage supplied by phase A driver so as to induce the commanded current $i_A = 4i_{mA}$ on phase A.

u_B = Voltage supplied by phase B driver so as to induce the commanded current $i_B = 4i_{mB}$ on phase B.

M_{PM} = Magnetomotive force of the permanent magnet=1547A

R_{PM} = Reluctance of the permanent magnet = 5707000 H^{-1}

R_{DW} = Driver internal resistance + module winding resistance = 31 Ω

p = platen pitch = motor pitch= 0.001016 m

Note that the Normag two-dimensional linear electric motor features four windings per phase per axis. Therefore $i_A = 4i_{mA}$ and $i_B = 4i_{mB}$ for axis X and Y.

ii. Model Simulation and Comparison with Plant Output

The fourth order model predicts to a very high degree the behavior of the real system when the usual test trajectories are taken as inputs. In this section a comparison of the model simulation output and the real plant output is discussed.

Simulink® was used to simulate the response of the system and test the model with different inputs. For the present analysis, all the test trajectories selected were confined to be one-dimensional trajectories, i.e. only along the X-axis. As it is seen from Figure 2, the behavior of the plant for this kind of trajectories can be reasonably predicted by the fourth order model. The only difference which can be detected is the higher frequency of the oscillations around the limits of the trajectory on the simulation than those on the real plant measurement.

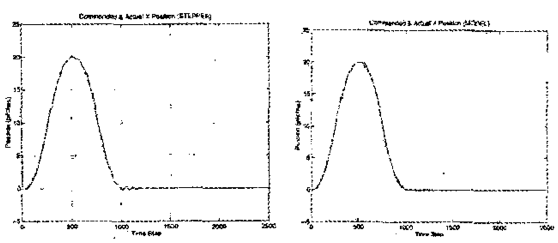

Fig. 2. Plant output and fourth order model simulation.

B. Second Order Model

In this section the equations describing a reduced second order model are presented. The simplification is motivated by the desire to model the linear motor plant by means of a much simpler representation than the one presented in the preceding section. The simplification introduced here consists on the elimination of the current driver dynamics. Each one of the four drivers required to energize a single linear motor introduces two new state variables to our system. Because it is assumed that all the drivers are calibrated so as to have exactly the same DC gain and the same settling time, it is reasonable to consider only two total states to be introduced by the dynamics of the four drivers.

These drivers are the UD-12 model from Parker Compumotor which is a regulated current pulse-width modulated power amplifier. The input to the driver is the commanded current. For the Normag linear motor, the maximum current is set to 4 amps. The driver operates as an effort source, it is to say; the output of the driver at any time is the voltage required to induce the commanded magnetizing current through the winding. This amplifier, or driver, incorporates an internal loop to ensure that the output current tends to the commanded value. The bandwidth of these drivers is greater than 2500 Hz, with a switching frequency of 20000 Hz. The transfer function describing the dynamics of each amplifier is [3, 7]:

$$F_d(s) = \frac{230000s + 256000000}{2.216s^2 + 46316s} \tag{20}$$

Obviously, the justification for the elimination of the driver dynamics from the linear motor model assumes that the drivers can be modeled as constant gains.

i. Underlying Equations

If u is the commanded position, then the plant under open loop control (stepper control) can be modeled as a nonlinear second order system. Because the amplifier dynamics is assumed to be a constant gain:

$$i_{mA} = I_0 \sin\left(\frac{2\pi u}{p}\right) \qquad i_{mB} = I_0 \cos\left(\frac{2\pi u}{p}\right) \tag{21}$$

The air gap reluctances are calculated as before, but the expressions for the fluxes through the pole faces are simplified to:

$$\phi_1 = \frac{R_2 M_{PM} - (R_2 + R_{PM})ni_{mA}}{2R_1R_2 \quad (R_1 \quad R_2)R_{PM}} \tag{22}$$

$$\phi_2 = \frac{R_1 M_{PM} + (R_1 + R_{PM})ni_{mA}}{2R_1R_2 \quad (R_1 \quad R_2)R_{PM}} \tag{23}$$

$$\phi_5 = \frac{R_6 M_{PM} - (R_6 + R_{PM})ni_{mA}}{2R_5R_6 \quad (R_5 \quad R_6)R_{PM}} \tag{24}$$

$$\phi_6 = \frac{R_5 M_{PM} + (R_5 + R_{PM})ni_{mA}}{2R_5R_6 \quad (R_5 \quad R_6)R_{PM}} \tag{25}$$

The force equations do not change. It is to say, equation (16) is still valid as well as equation (20) for the drag force.

ii. Model Simulation and Comparison with Plant Output

Again, a block diagram for the new model was constructed and the response to the same input commented above was simulated (see Figure 3).

As it is seen, this model does not predict the behavior of the plant as the fourth order model does. The main difference is on the amplitude of the oscillations about the commanded trajectory. However, this nonlinear model can be simplified and latter linearized by feedback liberalization, as it is explained in the next sections. It was decided that the adaptive controller should be developed on this simple model and latter tested on the more complex and detailed models.

C. Approximated Model

In this section an approximation of the second order model is introduced. As it is seen below, the result is still a nonlinear model. Fortunately, the expression reached is much simpler and suitable for feedback linearization.

i. Underlying Equations

If the entire set of equations describing the dynamics considered on the second order model are studied carefully, it can be concluded that the force generated (without drag force) is a function of commanded and actual positions only. This statement agrees with the result of a previous work, which modeled the linear motor as a spring-mass system. Therefore, the force generation surface was studied and approximated to:

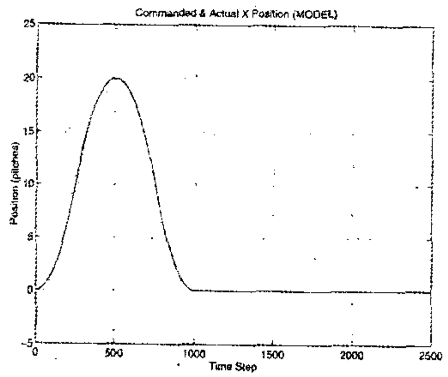

Fig. 3. Second order model simulation.

$$F = A\sin\left(\frac{2\pi(u-x)}{p}\right) \tag{26}$$

Where u is the commanded position, x the actual position and A is a coefficient, which was set to 460 to minimize the error between the model and the approximation.

From previous results [2, 3] it is known that the linear electric motor can be driven in at least two different ways: current control and commanded position control (lead angle control). Both parameters, current and lead angle, have a direct influence on the force generation and therefore on the stiffness, disturbance rejection and maximum acceleration of the motor.

Current control refers to the regulation of the current amplitude. Lead angle control allows us to command a position (u in previous equations) so as to produce a desired lead angle for a static or a dynamic state.

The lead angle can be understood as the difference at any time between the actual position and the commanded position. This parameter has to be non-zero in order to generate a non-equilibrium state and, therefore, to generate forces and induce movement. That is the motivation for a simple linear motor model based on a spring-mass configuration. However, because of the cyclic nature of the teeth of the motor and the grooves of the platen, the force generated and the lead angle relation is non-linear. In fact, it is a crucial parameter for the linear motor control. It is desired to know the required lead angle for maximum force generation under all operating conditions.

It is believed that, below the limit where the nonlinear effects become dominant, the force generation is almost directly proportional to the magnitude of the lead angle. It is to say, in this region, the system can be modeled as a mass-spring system. One problem from the implementation point of view is that if a constant generation force is desired along a trajectory then the lead angle has to be kept at the respective value, even in the presence of delays, disturbances and losses. Current amplitude, on the other hand, can be considered to be directly proportional to the force generated. Obviously, the force generation in this case is also affected by the operating condition. In general, the force generated at zero speed is higher than the force generated at non-zero speeds. As a general consequence of the details commented above, our objective now should be to find the best control variable that gives us good controllability. In this paper only the commanded position control was studied. Finally, it is desired to linearize the drag force. If the drag force is linearized in v (velocity) then it is very easy to incorporate it in the state space representation, because even the simplest second order model takes position and velocity as state variables.

Therefore, it was decided to approximate the equation describing the drag force (equation (20)) to a linear formula. For the range of operation of the motor, the following approximation introduces another error; in this case, the maximum error is smaller than 0.5%.

$$F_d = 53.62v \tag{27}$$

When this simplified model is used, the following result is obtained: Figure 4 shows the surfaces representing the error introduced by this simplification. If A = 460, the error at any point is smaller than 3%

III. Full State Feedback Adaptive Control

In this section, a very simple structure of the linear motor model is presented. Most of the adaptive controller development is based on this structure. The controller is then simulated and the convergence of the parameters is commented. Then, input linearization is applied in order to study the behavior of the adaptive controller on the second and fourth order models. Very important issues related to robustness are introduced at this point.

A. Structure of the System to be Controlled

If the simplification on the force generated and the simplification on the drag force are introduced into the expression for the net force, then one obtains:

$$F_{net} = 460\sin\left(\frac{2\pi(u-x)}{p}\right) \quad 53.62v \quad d(u,x) \tag{28}$$

Where $d(u,x)$ is a bounded disturbance, if no external disturbance is considered. It is because the errors from the approximations have been proved to be small. Besides, it is important to make clear here that the total error as a function of the states (total error = error in force generation + error in drag force computation) cannot be computed from any combination of signals of the linearized model.

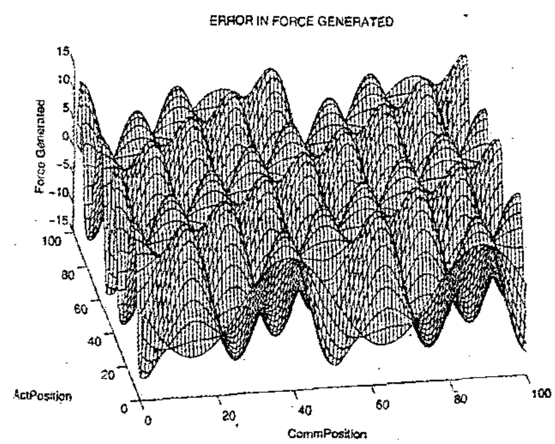

Fig. 4. Difference in force generation between second order model and approximated model.

With all these considerations, the structure of a linear plant for our problem is of the form presented in Figure 5. This structure incorporates input linearization. The drag force is included in the state space representation and the adaptive controller should be able to estimate the true value of this coefficient so as to improve tracking. Full state feedback (state variables accessible), PID control or phase-lead control seems to be suitable for this problem.

After the commanded force generation has been computed, it is required to calculate the commanded position. This task is done by computing u with the inverse of equation (26), where F is the output of the controller and x is the position signal.

Once the commanded position is computed, this signal is feed into the second order model, the fourth order model or the real plant. However, for the purpose of developing the adaptive control it is assumed that the input linearization cancels the part of the model corresponding to $F(x,u)$, arriving to the very simple and linear structure presented in Figure 5.

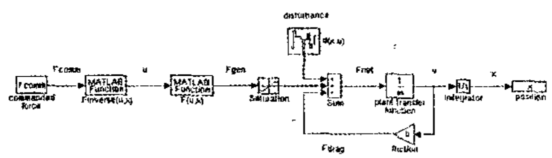

Fig. 5. Input linearization of the linear motor plant.

B. Adaptive Control of the Linear Motor

Full State Feedback Adaptive Control was chosen as a first implementation of the controller for the force generation. This decision was made based on a desire of keeping the initial solution as simple as possible.

The controller to be developed should be robust against the errors introduced by the simplification of the force generated and the drag force as well as the not well known parameters involved: the effective mass (m) and the effective friction coefficient (b). Because the order of the system is two, the adaptive controller has to estimate three parameters. The adaptive control structure's schematic description can be seen in Figure 6. In this figure subscript $_M$ refers to the model and subscript $_p$ refers to the plant. After the simulations, the force-parameter history and tracking error of in Figures 7 and 8 is obtained.

The reference input chosen to be a sequence of the same reference trajectory described above. It is not a sinusoid, because it is the position profile resulting from a succession of periods of constant acceleration and constant deceleration. Therefore, this reference signal is not persistently exciting and the parameters being estimated may not converge to the true values. After simple derivations it is calculated that the required gains so as to see convergence in about 4 seconds with no noisy signals were found to be:

$$K_{v_{A1}} = 100000 \quad K_{v_{A2}} = 10000 \quad K_{v_K} = 100000$$

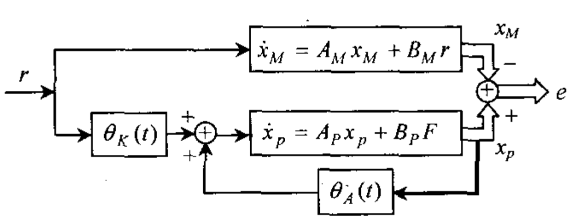

Fig. 6. Schematic of adaptive control structure.

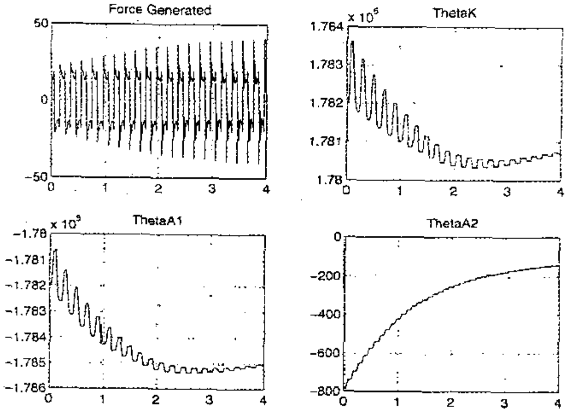

Fig. 7. Force and parameters history, adaptive control of the linear plant.

IV. CONCLUSIONS

In this paper, the development of an adaptive control for a two-dimensional linear motor for robotic palletizing systems is presented. For this development, first the modeling of the linear motor has been com pleted. After deciding on the best model, an adaptive controller has been suggested to improve the reliability of the system.

From the results it is concluded that an adaptive controller can be implemented for the control of the linear motor plant. But this controller may need to be improved if the application of the machine based on linear motors requires accurate tracking. If the application calls for maximum accelerations, maximum velocities and/or strong disturbance rejections then we need to ensure a proper response from the motor since the very beginning of the execution, in order to avoid loss of synchronism. Furthermore, usual trajectories for this machine are not persistently exciting.

However, an adaptive controller could be used to estimate the parameters of a fixed (non-adaptive) controller at the beginning of the execution in order to find the optimum values corresponding to the operating conditions of the plant at that time. This method seems to be very realistic due to the time varying operating conditions of most mechanical systems, a phenomenon generally not included in the model.

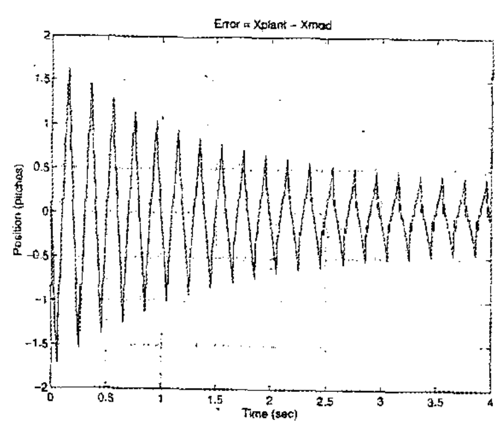

Fig. 8. Tracking error, adaptive control of the linear plant.

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