VELOCITY FEEDBACK DYNAMIC VIBRATION CONTROLLER SYNTHESIS: LMI APPROACH

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Abstract— **In this paper, a linear matrix inequality (LMI) approach combined with cone complementarity algorithm is presented for synthesis of H_∞ velocity feedback vibration controllers. The full/reduced order dynamic controllers are considered to be decentralized and positive real. Linear matrix inequalities (LMIs) are used to form these constraints. Examples are presented to demonstrate the approach.**

I. INTRODUCTION

High amplitude values in vibration systems are usually undesired because of their negative effects on performance and safety of the system. These peak values can be eliminated using various control approaches [1], [2], [3]. One of these methods is the velocity feedback control [8]. In this method, velocity values of the system is used to create the control action, which can either be in static or dynamic state. This paper treats the case of dynamic output feedback.

The decision of selecting the controller parameters depends on the performance requirements of both the controller and the uncontrolled system. The bounded real lemma for the closed loop system is the starting point for the formulation of desired system specifications. The constraints on the controller transfer function can be considered to be decentralized and positive real. The decentralized controller structure is in a diagonal or block diagonal form, thus the input/output pairing can be established. On the other hand, the positive realness is the key criteria in order to design passive controllers [4].

Linear matrix inequalities are used to represent these constraints on system and controller performances [5], [6], [7]. In this paper; a combined LMI-cone complementarity algorithm is used in order to compute the velocity feedback controller parameters.

II. H_∞ CONTROL

Consider a n_p^{th} order linear time-invariant generalized plant P containing the vibrating plant and all frequency-dependent weighting functions. The plant transfer function can be written in state space form as

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} x \\ w \\ u \end{bmatrix}. \qquad (1)$$

where the matrices are arranged in the form;

$$\begin{aligned} B_2 &= \begin{bmatrix} B_{21} & B_{22} & \cdots & B_{2N} \end{bmatrix} \\ D_{12} &= \begin{bmatrix} D_{121} & D_{122} & \cdots & D_{12N} \end{bmatrix} \\ C_2 &= \begin{bmatrix} C_{21} & C_{22} & \cdots & C_{2N} \end{bmatrix}^T \\ D_{21} &= \begin{bmatrix} D_{211} & D_{212} & \cdots & D_{21N} \end{bmatrix}^T \\ D_{22} &= \begin{bmatrix} D_{221} & D_{222} & \cdots & D_{22N} \end{bmatrix} \end{aligned}$$

and $i = 1, .., N$, total number of controller forces acting on the plant.

The aim is to minimize the vibration amplitude vector, $z(t) \in R^{n_z}$. $y_i(t) \in R^{n_y}$ and $u_i(t) \in R^{n_u}$ are the ith observation vector representing the measured variables, here velocities, and corresponding ith control input vector, respectively. $x(t) \in R^{n_p}$ is the state vector of the system. The disturbance vector $w(t) \in R^{n_w}$ contains all external inputs, including disturbances, sensor noise, and commands. The matrices $A, B_{91}, B_{72}, C_{11}, D_{11}, D_{12}, C_{21}, D_{21}, D_{22}$ are constant and compatible in dimension with corresponding vectors.

The equations of motion for linear time-invariant dynamic controllers of fixed order n_c are given as:

$$\begin{aligned} \dot{x}_c(t) &= A_c x_c(t) + B_c y(t) \\ u(t) &= C_c x_c(t) + D_c y(t) \end{aligned} \qquad (2)$$

arranging in the matrix form:

$$\begin{bmatrix} \dot{x}_c \\ u \end{bmatrix} = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \begin{bmatrix} x_c \\ y \end{bmatrix} \qquad (3)$$

where $x_c \in R^{n_c}$ is the controller state.

The controller transfer function matrix is

$$K = \begin{bmatrix} A_c & B_c \\ C_c & D_c \end{bmatrix} \qquad (4)$$

When a linear controller with transfer function $K(s)$ inserted from y to u, the closed loop transfer function from w to z can be constructed (Figure 1). If the open-loop system is augmented with the states corresponding to the controller, the following augmented system can be obtained:

$$\begin{bmatrix} \dot{x} \\ \dot{x}_c \\ z \\ x_c \\ y \end{bmatrix} = \begin{bmatrix} A & 0 & B_1 & 0 & B_2 \\ 0 & 0 & 0 & I_{n_c} & 0 \\ C_1 & 0 & D_{11} & 0 & D_{12} \\ 0 & I_{n_c} & 0 & 0 & 0 \\ C_0 & 0 & D_{29} & 0 & D_{21} \end{bmatrix} \begin{bmatrix} x \\ x_c \\ w \\ \dot{x}_c \\ u \end{bmatrix} \qquad (5)$$

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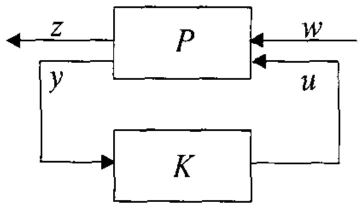

Fig. 1. Generalized plant-controller configuration

equivalently,

$$\begin{bmatrix} \dot{\tilde{x}} \\ z \\ \tilde{y} \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}_1 & \tilde{B}_2 \\ \tilde{C}_1 & \tilde{D}_{11} & \tilde{D}_{12} \\ \tilde{C}_2 & \tilde{D}_{21} & \tilde{D}_{52} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ w \\ \tilde{u} \end{bmatrix}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ x_c \end{bmatrix}, \tilde{y} = \begin{bmatrix} x_c \\ y \end{bmatrix}, \tilde{u} = \begin{bmatrix} \dot{x}_c \\ u \end{bmatrix}$$

with

$$\tilde{u} = K\tilde{y}$$

$$\begin{bmatrix} \dot{\tilde{x}} \\ z \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_{11} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ w \end{bmatrix}.$$

The closed-loop system matrix can be written as an affine function of the controller matrix as follows:

$$\begin{bmatrix} \bar{A} & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_{11} \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}_1 \\ \tilde{C}_1 & \tilde{D}_{11} \end{bmatrix} + \begin{bmatrix} \tilde{B}_2 \\ \tilde{D}_{12} \end{bmatrix} \check{K} \begin{bmatrix} \tilde{C}_2 & \tilde{D}_{21} \end{bmatrix} \quad (6)$$

where

$$\check{K} = \begin{bmatrix} A_c + B_c(I - D_{22}D_c)^{-1}D_{22}C_c & B_c(I - D_{22}D_c)^{-1} \\ C_c(I - D_{22}D_c)^{-1} & D_c(I - D_{22}D_c)^{-1} \end{bmatrix} \quad (7)$$

$\|T_{zw}\|_\infty$ denotes H_∞ norm of the closed-loop transfer function from w to z where $T_{zw} = \bar{D}_{11} + \bar{C}_1(sI - \bar{A})^{-1}\bar{B}_1$, i.e. its largest gain across frequency in the singular value norm. $\|T_{zw}\|_\infty < \gamma$ can be interpreted as a disturbance rejection performance, so the following lemma can be introduced:

Lemma 1: **Bounded Real Lemma** [5] Given a system of the form

$$\begin{bmatrix} \dot{\tilde{x}} \\ z \end{bmatrix} = \begin{bmatrix} \bar{A} & \bar{B}_1 \\ \bar{C}_1 & \bar{D}_{11} \end{bmatrix} \begin{bmatrix} \tilde{x} \\ w \end{bmatrix} \quad (8)$$

then the following statements are equivalent:

i) $\|T_{zw}(s)\|_\infty < \gamma$

ii) there exists a positive definite matrix Q such that

$$\begin{bmatrix} \bar{A}^T Q + Q\bar{A} & Q\bar{B}_1 & \bar{C}_1^T \\ \bar{B}_1^T Q & -\gamma I & \bar{D}_{11}^T \\ \bar{C}_1 & \bar{D}_{11} & -\gamma I \end{bmatrix} < 0. \quad (9)$$

Using the elimination Lemma [7] and following an algebraic procedure the following necessary and sufficient conditions for the H_∞ control problem can be obtained: There exists a controller that solves the fixed order H_∞ control problem if and only if there exist positive definite matrices X and Y such that

$$\begin{bmatrix} \begin{bmatrix} B_2 \\ D_{12} \end{bmatrix}^{\perp T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} AX + XA^T & XC_1^T & B_1 \\ C_1^T X & -\gamma I & D_{11} \\ B_1^T & D_{11}^T & -\gamma I \end{bmatrix} \begin{bmatrix} \begin{bmatrix} B_2 \\ D_{12} \end{bmatrix}^{\perp T} & 0 \\ 0 & I \end{bmatrix}^T < 0 \quad (10)$$

$$\begin{bmatrix} \begin{bmatrix} C_2^T \\ D_{21}^T \end{bmatrix}^{\perp T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A^T Y + YA & YB_1 & C_1^T \\ B_1^T Y & -\gamma I & D_{11}^T \\ C_1 & D_{12} & -\gamma I \end{bmatrix} \begin{bmatrix} \begin{bmatrix} C_2^T \\ D_{11}^T \end{bmatrix}^{\perp T} & 0 \\ 0 & I \end{bmatrix}^T < 0 \quad (11)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \geq 0 \quad (12)$$

$$rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \leq n_p + n_c \quad (13)$$

The rank constraint exists whenever the order of the controller is smaller than the order of the plant. The relation

$$Rank(I - XY) \leq n_c$$

can be written as

$$Rank \begin{bmatrix} X & I \\ I & Y \end{bmatrix}$$

$$= Rank \begin{bmatrix} I & 0 \\ -X^{-1} & I \end{bmatrix} \begin{bmatrix} X & I \\ I & Y \end{bmatrix} \begin{bmatrix} I & X^{-1} \\ 0 & I \end{bmatrix}.$$

$$= Rank \begin{bmatrix} X & 0 \\ 0 & Y - X^{-1} \end{bmatrix}.$$

$$\leq Rank(Y - X^{-1}) + Rank(X)$$

and it can be obtained that [5]

$Rank(X) = n_p, Rank(Y - X^{-1}) = Rank(Y_{12}Y_{22}^{-1}Y_{12}^T) \leq n_c$.

Then by introducing the notation

$$Q = \begin{bmatrix} Y & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, Q^{-1} = \begin{bmatrix} X & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \quad (14)$$

where $X, Y \in R^{n_p x n_p}$ and $X_{22}, Y_{22} \in R^{n_c x n_c}$ and inserting the expressions for the closed-loop matrices in the bounded real lemma condition, the following BMI formulation of the H_∞ control problem can be obtained: Find a parameter matrix $Q > 0$ and a controller matrix $\hat{K}$ such that

$$\begin{bmatrix} (\tilde{A} + \tilde{B}_2\hat{K}\tilde{C}_1)^T Q + Q(\tilde{A} + \tilde{B}_2\hat{K}\tilde{C}_2) & Q(\tilde{B}_1 + \tilde{B}_2\hat{K}\tilde{D}_{21}) & (\tilde{C}_1 + \tilde{D}_{12}\hat{K}\tilde{C}_2)^T \\ (\tilde{B}_1 + \tilde{B}_2\hat{K}\tilde{D}_{21})^T Q & -\gamma I & (\tilde{D}_{11} + \tilde{D}_{12}\hat{K}\tilde{D}_{21})^T \\ (\tilde{C}_8 + \tilde{D}_{12}\hat{K}\tilde{C}_2) & (\tilde{D}_{11} + \tilde{D}_{12}\hat{K}\tilde{D}_{21}) & -\gamma I \end{bmatrix} < 0 \quad (15)$$

(15) can be solved by standard LMI Matlab-Software.

III. CONSTRAINTS ON THE CONTROLLER

A. DECENTRALIZED CONTROLLER

For a decentralized controller with N-controller force action on the plant; the matrices A_c, B_c, C_c, D_c, consist of N submatrices $\hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i$, in following form:

$$\begin{aligned} A_c &= diag([\hat{A}_1]_{\hat{n}_1 \times \hat{n}_1}, [\hat{A}_2]_{\hat{n}_2 \times \hat{n}_2}, \cdots, [\hat{A}_N]_{\hat{n}_N \times \hat{n}_N})_{n_c \times n_c} \\ B_c &= diag([\hat{B}_1]_{\hat{n}_1 \times 1}, [\hat{B}_2]_{\hat{n}_2 \times 1}, \cdots, [\hat{B}_N]_{\hat{n}_N \times 1})_{n_c \times N} \\ C_c &= diag([\hat{C}_1]_{1 \times \hat{n}_1}, [\hat{C}_2]_{1 \times \hat{n}_2}, \cdots, [\hat{C}_N]_{1 \times \hat{n}_N})_{N \times n_c} \\ D_c &= diag([\hat{D}_1]_{1 \times 1}, [\hat{D}_2]_{1 \times 1}, \cdots, [\hat{D}_N]_{1 \times 1})_{N \times N} \end{aligned} \quad (16)$$

with

$$n_c = \sum_{i=1}^{N} \hat{n}_i \quad (17)$$

B. POSITIVE REALNESS

Lemma 2: **Positive Real Lemma** [5] The passivity property for positive realness of the controller is equivalent to the existence of any matrix $W = W^T > 0$ such that

$$\begin{bmatrix} A_c^T W + W A_c & C_c^T - W B_c \\ C_c - B_c^T W & -(D_c^T + D_c) \end{bmatrix} \leq 0. \quad (18)$$

IV. A COMBINED LMI-CONE COMPLEMENTARITY ALGORITHM

Before introducing the algorithm, the following sets in the space of symmetric matrices can be defined;

$\mathbb{D} = \{Z \in \mathbb{S}^{2n} : Z = \begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}, X, Y \in \mathbb{S}^n\}$,

$\mathbb{R}_k = \{Z \in \mathbb{S}^{2n} : rank(Z + J) \leq k\}$,

where $k = n_p + n_c$ and

$J = \begin{bmatrix} 0 & I_{n_p} \\ I_{n_p} & 0 \end{bmatrix} \in \mathbb{S}^{2n}$.

In addition to the convex LMI constraint sets(10 − 12), when the non-convex constraint exists, the following theorem should be used to compute the orthogonal projection onto the non-convex constraint set.

Theorem 1: [7] Let $Z \in \mathbb{S}^{2n}$ and let $Z + J = U\Sigma V^T$ be the singular value decomposition of $Z + J$. The orthogonal projection, $Z^* = P_{\mathbb{R}_k} Z$ onto the set $\mathbb{R}_k$ is given by

$$Z^* = U\Sigma_k V^T - J$$

where Σ_k is the diagonal matrix obtained by replacing the smallest $n_p - n_c$ singular values in $Z + J$ by zero.

Step 1: Find X, Y that satisfy the LMI constraints (10−12) and minimize γ. If the problem is infeasible, stop. Otherwise, $\gamma_{min} = \gamma$ and set $X_0 = X, Y_0 = Y$ and $k = 1$. Using (14), solve (15) for $\hat{K}$ and go to step 7. If the solution is infeasible, go to step 2.

Step 2: Set $\gamma_k = \gamma_{k-1} + \epsilon$ with $0 < \epsilon < 10e-2$. Find X_k, Y_k that solve the corresponding cone complementarity problem [6]:

Step 3: minimize $\mathbf{Tr}(X_{k-1}Y_k + X_k Y_{k-1})$ subject to LMI's $(10-12)$.

Step 4: If the objective $\mathbf{Tr}$ $(X_{k-1}Y_k + X_k Y_{k-1})$ has reached a stationary point, go to Step 5. Otherwise, set $k = k+1$ and go to Step 3.

Step 5: Denote the minimizing solutions by (X^*, Y^*); that is, the projection onto Γ_{convex} is written as $(X^*, Y^*) = P_{\Gamma_{convex}}(X_0, Y_0)$, construct Z. If there exist non-convex constraints apply the Theorem (1) and compute Z^*.

Step 6: Take $Q = Z$ or Z^* and solve the controller $\hat{K}$ in (15). If the solution is infeasible, go to step 2.

Step 7: When the positive realness constraint exists on the controller, check (18). If the controller satisfies (18), stop, else go to step 2.

V. NUMERICAL EXAMPLES

In this section a case study on controller synthesis is given. As a model, the two degree of freedom system [9] has the equations of motion

$$M\ddot{x} + D\dot{x} + K = \begin{bmatrix} F \\ 0 \end{bmatrix}$$

where

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0.02 & -0.01 \\ -0.01 & 0.01 \end{bmatrix}, K = \begin{bmatrix} 8 & -4 \\ -4 & 4 \end{bmatrix}$$

The external harmonic excitation force acts on the first degree of freedom. The ∞-norm of the second degree of freedom should be minimized. The controller acts on the second degree of freedom of the system. The corresponding state space matrices are;

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -8 & 4 & -0.02 & 0.01 \\ 4 & -4 & 0.01 & -0.01 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T, C_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

The standard assumptions for the system are:

A.1 $D_{11} = 0, D_{22} = 0$.

A.2 (A, B_1) is stabilizable and (C_1, A) is detectable.

A.3 (A, B_2) is stabilizable and (C_2, A) is detectable for existence of a stabilizing K.

A.4 For ensurance of proper and realizable controller : $rank D_{12} = n_u$, $rank D_{21} = n_y$.

A.5 $D_{12}^T \begin{bmatrix} C_1 & D_{12} \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$. It means that $C_1 x$ and $D_{12}u$ are orthogonal so that the penalty on $z = C_1 x + D_{12}u$ includes a nonsingular penalty on the control u.

A.6 $\begin{bmatrix} B_1 \\ D_{21} \end{bmatrix} D_{21}^T = \begin{bmatrix} 0 \\ I \end{bmatrix}$. It is dual to A.5 and concerns how the exogenous signal w enters P: w includes both plant disturbance and sensor noise, these are orthogonal, and the sensor noise weighting is nonsingular.

A.7 $rank \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n_p + n_u$ and $rank \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n_p + n_y \; \forall \omega \in R$. to ensure that the optimal controller does not try to cancel poles or zeros on the imaginary axis which would result in closed-loop instability.

A.8 The controller is assumed to be collocated;

$$C_2 = B_2^T$$

A. POSITIVE REAL FULL ORDER DYNAMIC CONTROLLER SYNTHESIS

A 4th order controller is to be synthesized. After the optimization ; the results are: $\|T_{zw}(s)\|_\infty < \gamma_{min} = 1.5572$. The minimized vibration amplitude of the second degree of freedom at is 0.4924. The frequency response with and without controller is given in Figure 2. The synthesized controller is :

$$K = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right] = \left[\begin{array}{cccc|c} -0.6773 & -1.3511 & 0.5425 & 7.0438 & 0.5373 \\ 0.6455 & -0.1479 & -1.2852 & -0.6603 & -0.1166 \\ 0.6579 & 0.6359 & -1.7561 & 5.7770 & -1.2922 \\ -0.8909 & 0.1488 & -0.3616 & -1.2904 & 0.6716 \\ \hline 0.3755 & 0.0775 & -0.9792 & 1.8920 & 0.0004 \end{array}\right]$$

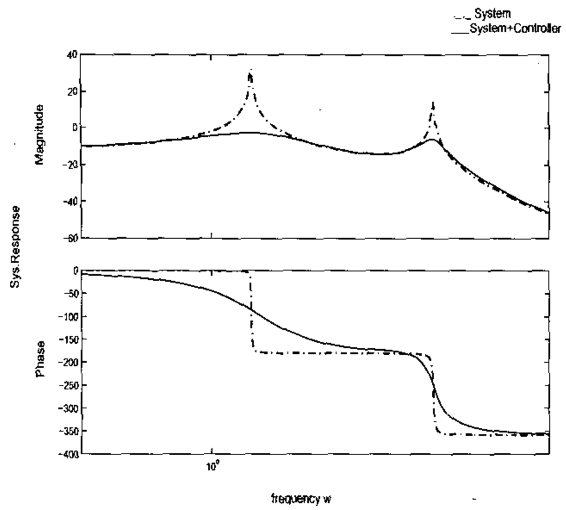

Fig. 2. Positive real full order controller introduction

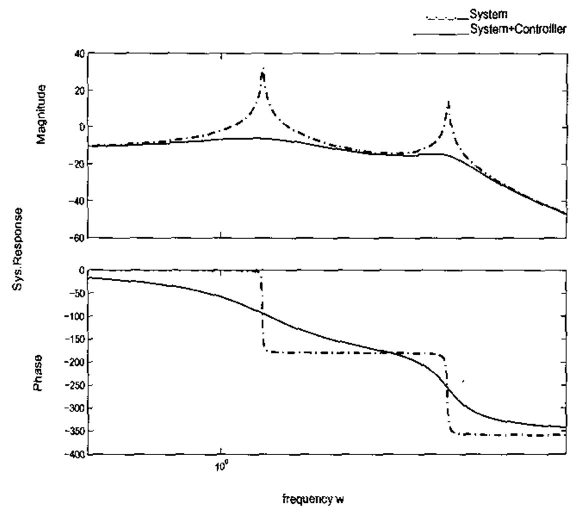

Fig. 4. positive real decentralized dynamic controller.

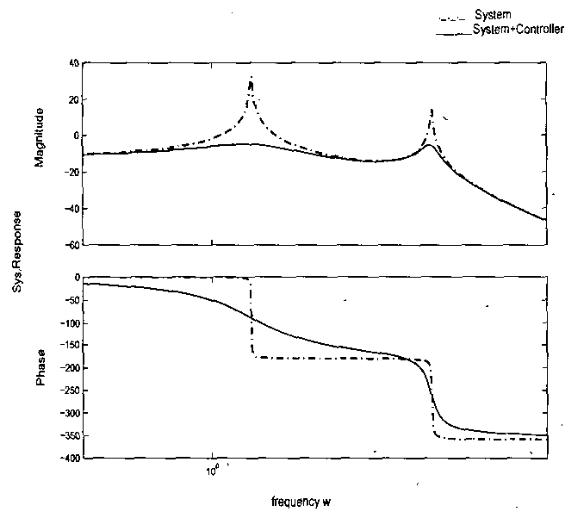

Fig. 3. Positive real reduced order dynamic controller introduction

B. POSITIVE REAL REDUCED ORDER DYNAMIC CONTROLLER SYNTHESIS

A 2nd order controller is to be synthesized. After the optimization; the results are: $\|T_{zw}(s)\|_\infty < \gamma_{min} = 1.6572$. The minimized vibration amplitude of the second degree of freedom is 0.5650. The frequency response with and without controller is given in Figure 3. The synthesized controller is :

$$K = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right] = \left[\begin{array}{cc|c} -8.6971 & -5.6155 & 0.0557 \\ -2.5912 & -29.7489 & 0.0259 \\ \hline 18.6986 & 11.6868 & 0.7609 \end{array}\right]$$

C. DECENTRALIZED DYNAMIC CONTROLLER SYNTHESIS

A 4th order decentralized controller is to be synthesized. After the optimization process; the results are: $\|T_{zw}(s)\|_\infty < \gamma_{min} = 2.0886$. The minimized vibration amplitude of the second degree of freedom at is 0.4924. The frequency response with and without controller is given in Figure 4. The synthesized controller is:

$$K = \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array}\right] = \left[\begin{array}{cccc|cc} -0.628 & 0.990 & 0 & 0 & -0.239 & 0 \\ -1.314 & -0.995 & 0 & 0 & -0.368 & 0 \\ 0 & 0 & -1.722 & 10.478 & 0 & 0.187 \\ 0 & 0 & -1.513 & -0.505 & 0 & -0.103 \\ \hline -0.266 & -0.344 & 0 & 0 & 0.560 & 0 \\ 0 & 0 & -0.047 & 0.572 & 0 & 0.837 \end{array}\right]$$

VI. CONCLUSION

In this paper, a solution method for the H_∞ control problem is presented using linear matrix inequalities (LMIs). Full, decentralized and reduced order positive real velocity feedback dynamic controllers are designed for this purpose. The constraints on the system and controller transfer functions increase the H_∞ norm and give less effective results for the system performance.

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